Math 20E | Second Midterm | 60 points | May 26, 2004

- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  One side of one page of NOTES is allowed.
- **You must show your work to receive credit.**

1. (18 points) Fill in the blanks with either functions, numbers or words. You need not copy the statements — just write what goes in the blanks. Some examples of words you might or might not use are

   harmonic irrotational potential scalar solenoidal vector
   curl divergence gradient laplacian

   (a) If $\nabla \times \mathbf{F} = \nabla \times \mathbf{G}$ for all $\mathbf{R}$, then $\mathbf{F} - \mathbf{G}$ is the ____ of some ____ function.

   (b) If $\nabla^2 f(\mathbf{R}) = \nabla^2 g(\mathbf{R})$ for $|\mathbf{R}| \le 1$ and $f = g$ for $|\mathbf{R}| = 1$, then $f(\mathbf{R}) - g(\mathbf{R}) =$____ for $|\mathbf{R}| \le 1$.

   (c) If $\nabla^2 f = 0$, we call f a (or an) ____ function.

   (d) The fundamental theorem of vector analysis states that a nice vector function in a nice domain can be written as the sum of a (or an) ____ and a (or an) ____.

2. (10 points) Compute $\iint_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS$ where $\mathbf{F}(x, y, z) = ze^{xy}\mathbf{i}$ and S is the ellipsoid $x^2 + 2y^2 + 3z^2 = 1$.

3. (20 points) The function $\mathbf{F} = 24xz\mathbf{i} - 12z^2\mathbf{k}$ has zero divergence.

   (a) Find a vector potential for $\mathbf{F}$; that is, find $\mathbf{G}$ whose curl is $\mathbf{F}$.

   (b) Find a vector potential for $\mathbf{F}$ that has no $\mathbf{k}$ component.
   (This may or may not be the same as your answer to (a).)

4. (12 points) Let D be the region in the xy-plane where $|x| + |y| \le 1$. In other words, it is the region given by

$$-1 \le x + y \le 1 \quad \text{and} \quad -1 \le x - y \le 1.$$

   Rewrite $\iint_D (x - y)^2\, e^{x^2 - y^2}\, dx\, dy$ as an integral over u and v by using the substitution $u = x + y$, $v = x - y$. Remember to describe the domain of integration.
   Remark: The (u, v) integral can be evaluated by integrating over u and then v; however, you are **NOT** being asked to evaluate it.

END OF EXAM