1. Since curl, divergence and cross product require that the functions be vectors, (a), (b) and (d) make no sense.
   (c) zero since the cross product is perpendicular to **F** and the dot product of perpendicular vectors is zero.
   (e) $\nabla \cdot \mathbf{F} \;=\; 2x + 0 + 1 \;=\; 2x + 1$.
   (f) zero since the cross product of a vector with itself is zero.

2. (a) is a ball with a circular hole. It is open and connected but not simply connected.
   (b) is a cylinder with a ball removed. It is open, connected and simply connected.
   (c) is the xy-plane with the vertical strip $-1 < x < 1$ removed. It is not open, not connected and (therefore) not simply connected.

3. We could try to find f with $\nabla f = \mathbf{F}$, but first we should check the equations $\partial F_1/\partial y = \partial F_2/\partial x$ $\partial F_1/\partial z = \partial F_3/\partial x$ and $\partial F_2/\partial z = \partial F_3/\partial y$. Since they fail, the vector field is not conservative.

4. We can parameterize the line segment by $\mathbf{R} \;=\; t\mathbf{Q} + (1-t)\mathbf{P} \;=\; (1-t,\ 1+t,\ t)$ for $0 \le t \le 1$. Then $d\mathbf{R}/dt \;=\; (-1,1,1)$ and $\mathbf{F}(\mathbf{R}) \;=\; (1-t,\ 1-t^2,\ 0)$. Thus the answer is

$$\int_0^1 (1-t,\ 1-t^2,\ 0)\cdot(-1,1,1)\ dt \;=\; \int_0^1 ((t-1)+(1-t^2))\ dt \;=\; 1^2/2 - 1^3/3 \;=\; 1/6.$$

5. Since the field is conservative and we are integrating around a closed curve, the integral is zero.