- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  One side of one page of NOTES is allowed.
- **You must show your work to receive credit.**
- Hint: In some cases, short cuts can reduce computation.

1. (24 points) Let $\mathbf{F} = (x^2 + y^2)\mathbf{i} + z\mathbf{j} + z\mathbf{k}$, and $f = xyz$. In each case, either compute the requested item or briefly explain why it makes no sense.

   (a) the curl of f (b) the divergence of f (c) $(\mathbf{F} \times \nabla f) \cdot \mathbf{F}$

   (d) $f \times \mathbf{F}$ (e) the divergence of $\mathbf{F}$ (f) $(\mathbf{F} \times \mathbf{F}) \times \mathbf{F}$

2. (9 points) For each of the following regions, answer the following questions:

   (i) Is it open? (ii) Is it connected? (iii) Is it simply connected?

   You do not need to explain your answers.

   (a) The set of points (x, y, z) with $x^2 + y^2 + z^2 < 2$ and $x^2 + y^2 > 1$.

   (b) The set of points (x, y, z) with $x^2 + y^2 < 2$ and $x^2 + y^2 + z^2 > 1$.

   (c) The set of points (x, y) with $|x| \geq 1$.

3. (5 points) Prove or disprove: The vector field defined by $\mathbf{F} = xyz\mathbf{i} + xz\mathbf{j} + y\mathbf{k}$ for all (x, y, z) is conservative.

4. (5 points) Compute $\int_C \mathbf{F} \cdot d\mathbf{R}$ where $\mathbf{F} = x\mathbf{i} + xy\mathbf{j}$ and C is the straight line segment from the point with position vector $\mathbf{P} = (1, 1, 0)$ to the point with position vector $\mathbf{Q} = (0, 2, 1)$.

5. (5 points) Let $f(x, y, z) = \tan(\sin(xe^y) + z^2)$ and let $\mathbf{F} = \nabla f$. Compute $\int_C \mathbf{F} \cdot d\mathbf{R}$ where C is the closed curve given by $(x, y, z) = (\cos t, \sin t, e^t \sin t)$, $0 \leq t \leq 2\pi$.

END OF EXAM