- Please put your name, ID number, and section number (or time) on your blue book.
- The first page of your blue book may contain notes. No other paper is allowed.
- **Calculators are NOT allowed.**
- You must show your work to receive credit.

1. (20 pts) Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(x-2)^n}{n+1}$.
   (a) For what values of x is the series absolutely convergent? Your answer should be an interval for x; that is, an expression like $a \le x < b$, or $a < x < b$, etc.
   (b) For what values of x is the series conditionally convergent?

2. (60 pts) Solve the following differential equations. If initial conditions are given, find the particular solution. If there are no initial conditions, find the general solution.
   (a) $y' = \dfrac{2x-y}{x+1}$; $y(0) = 2$.
   (b) $x^2y'' + 3xy' - 3y = 0$; $y(1) = 4$, $y'(1) = 0$.
   (c) $y'' - 4y = 16 \ln t$. You may leave integrals in your answer to (c).

3. (20 pts) Consider the differential equation $y'(t) = \sin(y(t))$.
   (a) Find a *stable* equilibrium point for the equation.
   (b) Find an *unstable* equilibrium point for the equation.

4. (20 pts) Find the first four nonzero terms of the series solution to the initial value problem
$$y'' - xy' - 2y = 0; \quad y(0) = 1, \quad y'(0) = 0.$$

5. (20 pts) One solution of the differential equation
$$t^3y'' - ty' + y = 0$$
is $y(t) = t$. Use reduction of order to find the general solution when $t > 0$.

6. (15 pts) Find the singular points of $x^2(1-x^2)y'' + y' + xy = 0$. Determine which are regular and which are not.

7. (20 pts) Find the Laplace transform of the solution to
$$y'' - 2y' + y = g(t); \quad y(0) = 1, \quad y'(0) = 0,$$
where
$$g(t) = \begin{cases} 1 & \text{for } 0 \le t \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

That's all folks!