Bender Math 21D First Midterm 10/25/99

- Please put your name, ID number, and section number (or time) on your blue book.
- The first page of your blue book may contain notes. No other paper is allowed.
- **You must show your work to receive credit.**
  "My calculator says ..." does NOT "show your work"!

1. (50 pts.) Determine if each of the following series is convergent or divergent.
   You must give correct reasons for your answers to receive credit.

   (a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$ (b) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n \ln n}$ (c) $\displaystyle\sum_{n=0}^{\infty} \frac{n^9 + 100\cos n}{\sqrt{n^3 + e^n}}$

   (d) $\displaystyle\sum_{n=2}^{\infty} \frac{\ln n}{n^2}$ (e) $\displaystyle\sum_{n=1}^{\infty} \cos n$

2. (15 pts.) Suppose that $\{a_n\}_{n=1}^{\infty}$ is a sequence of numbers and consider the series $S = \displaystyle\sum_{n=1}^{\infty} (a_{n+2} - a_n)$. Let S_N be the partial sum of the first N terms.

   (a) Write out S_3, S_4, and S_5 in as simple terms as you can. Use these to deduce what S_N is for general N.

   (b) Suppose $\lim_{n\to\infty} a_n$ exists and call the limit L. Show that S exists and obtain a simple formula for it. ("Simple formula" means a small finite expression involving possibly L and some a_i's.)

3. (15 pts) Find the radius of convergence AND interval of convergence of the power series $\displaystyle\sum_{n=0}^{\infty} \frac{n^2 - 2n}{3^n}(x-5)^n$.

4. (10 pts) Find the Taylor series for $\cos x$ at $a = \pi/4$. (You need not prove that $R_n(x) \to 0$.) Show your calculations!

5. (10 pts.) Let $f(x) = e^x \tan^{-1} x$. By Taylor's theorem, we know that

   $$f(x) = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + R_3(x),$$

   where $R_3(x)$ goes to zero like x^4 as x goes to zero.

   Find the coefficients c_0, c_1, c_2, and c_3. Show your calculations! *Hint.* If you multiply power series, you can avoid computing derivatives.

   For your information, $\tan^{-1} x = x - x^3/3 + x^5/5 \pm \cdots$.