

- Please put your name, ID number, and section number (or time) on your blue book.
- The first page of your blue book (both sides) may contain notes.
No other paper is allowed.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (20 pts) Suppose $(1 - x^2)^2 y'' + (1 - x)y' + (1 + x)y = 0$.
 - (a) Determine the singular points. Which ones are regular?
 - (b) Suppose we obtain a power series about $x_0 = 0$ for $y(x)$. Does the series converge at $x = 1/2$? You must give a *correct* reason for your answer to receive credit.

2. (15 pts) Compute the Laplace transform of $y(t)$ given that

$$y''(t) + y(t) = 1 + 2e^t, \quad y(0) = 1 \quad \text{and} \quad y'(0) = 2.$$

*Note that you are **not** asked to find $y(t)$.*

3. (10 pts) A cylindrical tank is 100 feet high and has a circular base of diameter 50 feet. A hole in the bottom of the tank allows water to leak out according to Torricelli's law: $dh/dt = -5\sqrt{h}$, where $h = h(t)$ is the depth of the water in feet and t is time in days. Water is pumped into the tank at a constant rate so that, if there were no hole, an empty tank would fill in 5 days. **Write down** a differential equation for $h(t)$ when *the tank starts out empty at $t = 0$* , taking into account the fact that the tank has a hole and water is being pumped in. Be sure to explain how you got the equation. *You need **not** solve the equation.*

4. (60 pts) Find the particular solutions to the following differential equations.

- (a) $xy' = 3x - 2y, \quad y(1) = 2.$

- (b) $xy' = (3x - 2)y, \quad y(1) = 2.$

- (c) $y'' - y = t, \quad y(0) = y'(0) = 0.$

- (d) $x^2 y' = x^2 - xy + y^2, \quad y(1) = 0.$

5. (20 pts) One solution to $x^3 y'' + xy' - y = 0$ is $y(x) = x$. Use reduction of order to find an independent solution for $x > 0$.

6. (20 pts) Find the power series solution about $x_0 = 0$ for the differential equation

$$(1 - x^2)y'' + 4y' + 6y = 0, \quad y(0) = 1, \quad y'(0) = -3.$$

You may use the following fact to help you check your calculations, *but you may **not** use it to find the solution:* The answer is a polynomial of rather low degree.

7. (30 pts) (a) Find the general solution to $2x^2 y'' + 3xy' - y = 0$ for $x > 0$.
(b) Find the general solution to $2x^2 y'' + 3xy' - y = 9x^2$ for $x > 0$.

That's all folks!