1. (a) This can be made into an exact equation with the integrating factor e^x, but it is easier to separate variables:

$$\int xe^x\,dx = -\int 2y\,dy \quad \text{and so} \quad xe^x - e^x = -y^2 + C,$$

where the first integral was done using the given formula with $a = n = 1$. Putting in $x = 0$ and $y = 2$, we obtain $C = 3$.

(b) This is a linear equation in $t(x)$: $xt'(x) + t = 3x^2$. The integrating factor is 1 and so $xt = x^3 + C$.

(c) This is exact since $\partial(ye^x)/\partial y = \partial(y + e^x)/\partial x$. Integrating gives $y^2/2 + ye^x = C$.

(d) The solution is $y(x) = 0$ for all x. Why not separate variables? You can separate variables provided $y \neq 0$. If we ignore the condition and proceed, we obtain

$$\int \frac{dy}{y^2} = \int 2x\,dx$$

and so $-1/y = x^2 + C$. Hence $y = \frac{-1}{x^2+C}$. Setting $x = y = 0$, we obtain $0 = -1/C$, which is impossible. Thus $C = \infty$, a second mistake since C must be a real number. With two mistakes, we can obtain the correct answer, but not credit for the problem.

(e) The roots of the characteristic equation are -1 and 5, so the general solution is $y = C_1e^{-t} + C_2e^{5t}$.

(f) The roots of the characteristic equation are $2 \pm i$, so the general solution is $y = (C_1 \cos t + C_2 \sin t)e^{2t}$. The initial conditions give $0 = C_1$ and $1 = 2C_1 + C_2$. Thus the particular solution is $y = e^{2t} \sin t$.

2. This problem can be done by solving the differential equation (separate variables). However, there is no need to do that much work.

(a) To find them, we solve $0 = y - y^3$, obtaining $y = -1, 0, 1$ for the three equilibrium points.

(b) Since $(y - y^3)' = 1 - 3y^2$, the function $y - y^3$ is decreasing at $y = \pm 1$ and increasing at $y = 0$. Thus $y = \pm 1$ are stable and $y = 0$ is unstable.

(c) The limit is 1. Since $y(0)$ is between the unstable equilibrium $y = 0$ and the stable equilibrium $y = 1$, $y(t)$ will move toward $y = 1$.