Bender Math 21D Outline of Solutions to First Midterm Feb. 3, 2000

1. Often more than one test can be used, so several alternative solutions are be given.
   (a) **Diverges**. Limit comparison test with $a_n = 1/n^{1/2}$, a divergent p-series. ($\lim_{n\to\infty} b_n/a_n = 1$)
   Also, integral test: $\int (x+3)^{-1/2}dx = 2(x+3)^{1/2} + C$.
   (b) **Converges**. Alternating series test: signs alternate, terms go to zero, absolute values of terms decrease.
   (c) **Diverges**. Limit comparison test with $a_n = 1/n$, a divergent p-series. ($\lim_{n\to\infty} b_n/a_n = 1$)
   Also, rewrite terms as $2^{-n} + 1/n$. Since $\sum 2^{-n}$ converges (geometric series) and $\sum 1/n$ diverges (p-series), the sum of the two diverges.
   (d) **Diverges**. The terms do not go to zero.
   (e) **Diverges**. Ratio test: $a_{n+1}/a_n = 6^2/3^3 = 36/27 > 1$.
   Also root test, or geometric series ($r = 6^2/3^3 > 1$), or the terms do not go to zero.
   (f) **Converges**. Same reasoning as (e): The ratio tests give $L = 27/36$ and the geometric series has $r = 27/36$. (Terms go to zero, so the last test mentioned in (e) cannot be used.)

2. Use the root test or ratio test to find R:

$$L = \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \frac{(n+1)^2|x+3|}{2n^2} = \frac{|x+3|}{2}.$$

This gives convergence for $|x+3| < 2$ and divergence for $|x+3| > 2$. Thus $R = 2$. (Alternatively, you can do what I mentioned in class: Replace $x+3$ with R and set $L = 1$: $1 = L = \lim \cdots = R/2$ and so $R = 2$.)

The endpoints of the interval are given by $x+3 = 2$ and $x+3 = -2$. The former gives the series $\sum n^2$ and the latter gives $\sum n^2(-1)^n$. Both diverge since the terms do not go to zero. Thus the interval of convergence is $-5 < x < -1$.

3. Since $e^x = \sum x^n/n!$, replacing x by $-2x^2$ gives

$$e^{-2x^2} = \sum_{n=0}^{\infty} \frac{(-2x^2)^n}{n!} = \sum_{n=0}^{\infty} \frac{(-2)^n x^{2n}}{n!}.$$

Multiply by $1+x$:

$$(1+x)e^{-2x^2} = \sum_{n=0}^{\infty} \frac{(-2)^n (x^{2n} + x^{2n+1})}{n!}.$$

The coefficient of x^{10} comes from the $n = 5$ term in the sum. The coefficient is $(-2)^5/5! = -4/15$. This is also the coefficient of x^{11}.