Math 20D | Final Exam | 100 points | June 12, 2007

- Print Name and ID number on your blue book.
- BOOKS and CALCULATORS are NOT allowed. Both sides of one page of NOTES is allowed.
- **You must show your work to receive credit.**

1. (60 pts.) Find the solution for each of the following differential equations. If initial conditions are given, find the particular solution. If no initial conditions are given, find the general solution.
   DO NOT LEAVE INTEGRALS IN YOUR ANSWERS.
   DO NOT LEAVE COMPLEX NUMBERS IN YOUR ANSWERS.

   (a) $y' + ty = t^3$ with $y(0) = 0$.

   (b) $t^2y' + e^y = 0$ on the interval $t > 0$.

   (c) $y'' - 2y' + 2y = 0$.

   (d) $y'' - 4y' + 4y = 2e^t$.

   (e) The system of two equations $x_1' = 2x_1 - x_2$ and $x_2' = 3x_1 - 2x_2$.

2. (18 pts.) This exercise deals with the power series solution for the differential equation $y'' - xy' - y = 0$ about $x_0 = 0$.

   (a) Find the recurrence relation for the coefficients of the power series solution.

   (b) Find the first six terms (i.e., up to and including the coefficient of x^5) of the particular solution that satisfies the initial conditions $y(0) = 1$ and $y'(0) = 2$.

3. (12 pts.) The motion of a simple rigid pendulum without friction can be described by the differential equation $d^2\theta/dt^2 = -K\sin\theta$ where $K > 0$ and θ is the angle the pendulum makes with the downward vertical.

   (a) Let $\omega = d\theta/dt$. Show that $d^2\theta/dt^2 = \omega\, d\omega/d\theta$.

   (b) It follows from (a) that the pendulum equation can be written $\omega\, d\omega/d\theta = -K\sin\theta$. Solve this equation.

4. (10 pts.) Given that

$$y'' - y = g(t), \quad y(0) = 1, \quad y'(0) = 2 \quad \text{and} \quad g(t) = \begin{cases} 1 - t & \text{for } 0 \le t \le 1 \\ 0 & \text{for } t \ge 1, \end{cases}$$

   find $Y(s)$, the Laplace transform of $y(t)$.

END OF EXAM