

- Please put your name and ID number on your blue book.
- CLOSED BOOK, but ONE SIDE of one page of notes are allowed.
- Calculators are NOT allowed.
- *In a multipart problem, you can do later parts without doing earlier ones.*
- **You must show your work to receive credit.**

1. (10 pts.) Here are some differential equations for the function y . For each equation (i) give its order and (ii) tell whether or not it is linear.

(a) $y''(t) = t^2y(t) + 7$ (b) $(x^2 + 1)dx = (x + 1)dy$ (c) $(y^2)' + y = 1$

(d) $y'y'' = 2$ (e) $x^2y'(x) + xy(x) + x^3 = 0$

2. (2 pts.) The functions $p(t)$ and $q(t)$ are continuous for all t and y_1 and y_2 are particular solutions to the linear homogeneous equation $y'' + p(t)y' + q(t)y = 0$. How can you tell if $c_1y_1 + c_2y_2$ is the general solution?

3. (6 pts.) Find the critical points (also called equilibrium points) of the autonomous differential equation $dy/dt = y(1 - y^2)$ and classify each one as asymptotically stable or unstable.

4. (32 pts.) Solve each of the following differential equations. If no initial conditions are given, find the general solution.

(a) $y'' + 9y = 0$; $y(0) = 0$, $y'(0) = 6$.

(b) $dx/dt = e^{x+t}$; $x(0) = 1$.

(c) $(2x + y)dx + (x - 2y)dy = 0$.

(d) $ty'(t) - y(t) = t^2$, $t > 0$.