Math 20D Final Exam 96 points 8AM Friday June 11, 2004

- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  Both sides of one page of NOTES is allowed.
- **You must show your work to receive credit.**

1. (10 points) It can be shown that $|\sin(x-1)| + |\sin x| > 1/2$ for all x. Using this fact, or otherwise, determine which of the following series converge and give reasons for your answers.

   (a) $\displaystyle\sum_{n=1}^{\infty} \frac{2|\sin n|}{n^2}$ (b) $\displaystyle\sum_{n=1}^{\infty} \frac{2|\sin n|}{n}$

2. (12 points) Consider the series $\displaystyle\sum_{n=0}^{\infty} \frac{(1-x)^n}{2n+1}$.

   (a) What is its radius of convergence?

   (b) For what values of x does it converge conditionally?

   (c) For what values of x does it converge absolutely?

3. (9 points per equation) Solve the following differential equations. If initial conditions are given, find the particular solution. If no initial conditions are given, find the general solution.

   (a) $dx = e^{x+t}dt$ with initial condition $x(0) = 0$.

   (b) $x'' - 2x' + 2x = 0$ with initial conditions $x(0) = 0$ and $x'(0) = 2$.

   (c) $(2x + y) + (3y^2 + x)y' = 0$.

   (d) $(z + w)dw = w\,dz$.

   (e) $x^2y'' - 6y = 5x^2$.

4. (5 points) A friend says that when he was taking notes in class the professor wrote a linear homogeneous second-order differential equation with the general solution $x = C_1t+C_2e^t$ for all t. He didn't copy the differential equation and wants help figuring it out. Explain why there <u>*cannot*</u> be such a differential equation $x''+p(t)x'+q(t)x = 0$.

THERE ARE MORE PROBLEMS

5. (8 points) A cylindrical tank is 16 feet high and has a circular base 5 feet in diameter. A small hole in the bottom allows water to leak out according to Torricelli's law: $dh/dt = -2\sqrt{h}$, where h is the depth of the water and t is time in hours. The tank starts out full of water.

   (a) How deep is the water in the tank after 1 hour?

   (b) How deep is the water in the tank after 6 hours?

6. (8 points) Suppose that the power series $y(x) = \sum_{n=0}^{\infty} a_n x^n$ is a solution to the differential equation $y'' - xy' - y = 0$. Find a recurrence relation for the a_n's and use it to compute a_3 and a_4 when the initial conditions are $y(0) = y'(0) = 1$.

7. (8 points) Find $Y(s)$, the Laplace transform of $y(t)$, given that

$$y'' + 2y = g(t), \quad y(0) = 0, \quad y'(0) = 1 \quad \text{and} \quad g(t) = \begin{cases} +1, & \text{for } 0 \leq t < 1, \\ -1, & \text{for } 1 \leq t < 2, \\ 0, & \text{for } t \geq 2. \end{cases}$$

END OF EXAM