1. (a) An integrating factor is $\exp\left(\int -\tan x\,dx\right) = \cos x$. Thus $(y\cos x)' = \cos x$ and

$$y(x) = \frac{1}{\cos x}\int \cos x\,dx = \frac{\sin x + C}{\cos x} = \tan x + C\sec x.$$

(b) Since the characteristic equation for the homogeneous equation is

$$0 = r^2 - 2r + 1 = (r-1)^2,$$

$y = C_1e^t + C_2te^t$ is the general solution to the homogeneous equation. By undetermined coefficients, a particular solution is $y = At + B$. Since $y' = A$ and $y'' = 0$, we have

$$4t = 0 - 2A + (At + B) = At + (B - 2A).$$

Thus $A = 4$ and $B = 8$. The general solution to (b) is therefore

$$y = C_1e^t + C_2te^t + 4t + 8.$$

(c) Rearrange, separate variables and integrate:

$$\int \frac{dy}{y} = \int \frac{dx}{1+x^2} \qquad \text{and so} \qquad \ln y = \arctan x + C.$$

You may leave the answer this way, with or without absolute values on y, or you may solve for y.

2. Since $4 + x^2 = 0$ for $x = \pm 2i$, the radius of convergence of the series for $\frac{1-x}{4+x^2}$ is 2. Thus the best we can guarantee is $|x| < 2$.

3. Since $y'' - 2y'/t + y/t^2 = e^t$ and $W[t, t^2] = \begin{vmatrix} t & t^2 \\ 1 & 2t \end{vmatrix} = t^2$, a particular solution is

$$y = -t\int \frac{t^2e^t}{t^2}dt + t^2\int \frac{te^t}{t^2}dt = -t\int e^t\,dt + t^2\int \frac{e^t\,dt}{t}.$$

4. By the table for Laplace transforms,

$$sY(s) + y(0) + e^{-s}Y(s) = 1/s \qquad \text{and so} \qquad Y(s) = \frac{1}{s(s+e^{-s})}.$$