Math 20D | Second Midterm | 48 points | May 26, 2004

- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  One side of one page of NOTES is allowed.
- **You must show your work to receive credit.**

Some integrals and derivatives that may be useful:

$\int \tan x\,dx = -\ln(\cos x) + C$ $\qquad (\tan x)' = \sec^2 x$ $\qquad (\sec x)' = \sec x\ \tan x$

$\int \sec x\,dx = \ln(\sec x + \tan x) + C$ $\qquad (\arctan x)' = \frac{1}{1+x^2}$ $\qquad (\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$

1. (27 points) Find the general solutions of the following differential equations.

   (a) $y' - (\tan x)y \;=\; 1$ (b) $y'' - 2y' + y \;=\; 4t$ (c) $(x^2+1)y' - y \;=\; 0.$

2. (3 points) I have decided to find a series solution $y = \sum a_n x^n$ to

$$(4 + x^2)y'' + (1 - x)y = 0.$$

   For what values of x can you guarantee that the series will converge? Why?
   You must give a reason—the "Why?"—**to receive credit.**

3. (9 points) $y = t$ and $y = t^2$ are solutions to $t^2y'' - 2ty' + 2y = 0$. Find a particular solution to $t^2y'' - 2ty' + 2y = t^2e^t$. You may leave integrals in your answer.

4. (9 points) $y(t)$ satisfies the differential equation

$$y'(t) + y(t-1) \;=\; 1,$$

   with $y(t) = 0$ for $t \le 0$. Compute the Laplace transform $Y(s) = \mathcal{L}\{y(t)\}$.

When your exam is returned, it will have the grade you will receive if you do NOT take the final exam. In computing that grade, this exam will be weighted more than the first exam to reflect the fact that the final will emphasize differential equations.

END OF EXAM