Math 20D First Midterm 48 points April 21, 2004

- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  One side of one page of NOTES is allowed.
- **You must show your work to receive credit.**

1. (5 points) Let $\sum a_n x^n$ be the Maclaurin series for $f(x) = (\cos x)\sin(x^2)$. Compute a_1, a_2, a_3 a_4 and a_5.

2. (15 points) Find the radii of convergence of the following power series. *You must give* ***correct*** *reasons for your answers to receive credit.*

$$\text{(a)} \sum_{n=0}^{\infty} \frac{x^n}{(2n)!} \qquad \text{(b)} \sum_{n=0}^{\infty} (2x-3)^n \qquad \text{(c)} \sum_{n=0}^{\infty} \frac{n^3 x^{n+1}}{n^2+1}$$

3. (20 points) Determine if each of the following series is convergent or divergent. If the series is convergent and the terms alternate in sign, determine the series is absolutely or conditionally convergent. *You must give* ***correct*** *reasons for your answers to receive credit.*

$$\text{(a)} \sum_{n=1}^{\infty} \frac{(-1)^n 3^{2n}}{n^3 2^{3n}} \qquad \text{(b)} \sum_{n=1}^{\infty} \frac{e^{-\sqrt{n}}}{\sqrt{n}} \qquad \text{(c)} \sum_{n=1}^{\infty} (-1)^n \sin(1/n) \qquad \text{(d)} \sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!}$$

4. (8 points) Let $\sum a_n x^n$ be the Maclaurin series for $f(x)$ and let $\sum b_n x^n$ be the Maclaurin series for $f(-x)$.
   (a) Express b_n in terms of a_n.
   (b) Suppose $f(x)$ is an even function; that is, $f(-x) = f(x)$. Show that, whenever n is odd, $a_n = 0$.

END OF EXAM