- Put your name, ID number, and section number (or time) on your blue book.
- You may have TWO PAGES of notes. NO CALCULATORS are allowed.
- **You must show your work to receive credit.**
- Please start each problem on a new page.

1. (15 pts) The equation $z = 4 - x^2 - y^2$ describes a surface. Write down an *iterated* integral for the area of that part of the surface that lies above the xy-plane. You need not evaluate the integral.

2. (20 pts) Change $(1, \sqrt{3}, 2)$ from rectangular coordinates to
   (a) cylindrical coordinates and
   (b) spherical coordinates.

   Your answers should give all angles exactly in radians and should *not* contain any inverse trig functions (that is, functions such as $\cos^{-1}$).

3. (20 pts) Here are two skew lines in parametric form:

$$\langle x(t), y(t), z(t)\rangle = \langle 1, 1, 0\rangle t,$$
$$\langle x(t), y(t), z(t)\rangle = \langle 0, 1, 2\rangle t + \langle 1, 1, 1\rangle.$$

   (a) Find a vector $\mathbf{v}$ that is perpendicular to both lines.
   (b) Compute the minimum distance between the lines.

4. (20 pts) Use Lagrange multipliers to find the maximum and minimum values of $f(x, y) = x^2 + 2y^2$ subject to the constraint $x^2 + y^2 = 1$. Also find the xy-coordinates of these extreme values.

5. (30 pts) There are two vector functions $\mathbf{f}(t)$ and $\mathbf{g}(t)$ which are defined and differentiable for all values of t. It is known that

$$f(2) = \langle 1, 0, 2 \rangle, \quad g(2) = \langle 0, 1, 1 \rangle, \quad f'(2) = \langle 1, 1, -1 \rangle, \quad \text{and} \quad g'(2) = \langle 1, -1, 0 \rangle.$$

For each of the following, compute a numerical value or explain why there is not enough information to do so. If your answer is a scalar, it should be written as a single number like 2, not $\mathbf{f}(2) \cdot \mathbf{g}(2)$; if a vector, it should be written in a form like $\langle 1, 2, 3 \rangle$.

(a) $(\mathbf{f}(t) \cdot \mathbf{g}(t))'$ at $t = 2$.

(b) $|\mathbf{f}(t)|'$ at $t = 2$.

(c) $(\mathbf{f}(t) \times \mathbf{f}(t))'$ at $t = 4$.

6. (20 pts) Sketch the region of integration and change the order of integration in

$$\int_0^2 \int_{x^2}^{2x} f(x, y)\, dy\, dx.$$

7. (15 pts) Suppose that $f(x, y)$ and $g(x, y)$ are continuous and differentiable and that $\langle f(x, y),\ g(x, y) \rangle$ is the gradient of some function $h(x, y)$. Prove that $f_y = g_x$.

8. (20 pts) The equation $z = 4 - x^2 - y^2$ describes a surface. Write down and evaluate a *polar coordinate* integral for the volume of the region that lies below the surface and above the xy-plane.

9. (20 pts) Compute $\displaystyle\iint_R (x + y)\, e^{xy}\, dA$ where $R = \{(x, y) \mid 0 \le x \le 1,\ 0 \le y \le 2\}$.

END OF EXAM