1. Use the chain rule.
   (a) $g_s = g_x\, x_s + g_y\, y_s = 2g_x + g_y$.
   (b) Let $h = g_s$. Then

$$\begin{aligned} g_{st} = g_{ts} = h_t &= h_x\, x_t + h_y\, y_t = h_x - h_y \\ &= (2g_{xx} + g_{xy}) - (2g_{xy} + g_{yy}) = 2g_{xx} - g_{xy} - g_{yy}. \end{aligned}$$

2. Note that $\nabla f = \langle 2x + 4y,\ 3y^2 + 2y + 4x\rangle$ and $\nabla f(0,1) = \langle 4, 5\rangle$.
   (a) $\mathbf{u} = \nabla f / |\nabla f| = 41^{-1/2}\langle 4, 5\rangle$.
   (b) The maximum is $|\nabla f| = \sqrt{41}$.
   (c) We need $\mathbf{u} \cdot \nabla f = 0$.
   There are two possible answers: $41^{-1/2}\langle 5, -4\rangle$ and $41^{-1/2}\langle -5, 4\rangle$.

3. This could be done it at least two ways.
   - The tangent line is in a direction in which $D_{\mathbf{u}} = 0$. Such a vector was found in 2(c). Then $\langle x, y\rangle = t\mathbf{u} + \langle 0, 1\rangle$. Since all we need is a vector parallel to $\mathbf{u}$, we can drop the factor of $41^{-1/2}$ if we wish to get the cleaner formula $\langle x, y\rangle = t\langle 5, -4\rangle + \langle 0, 1\rangle$.
   - Since $dy/dx = -f_x/f_y$, we have $dy/dx = -4/5$. Thus the line is $y - 1 = (-4/5)(x - 0)$, which can be written $y = -4x/5 + 1$.

4. (a) We need $\nabla f = \mathbf{0}$. By Problem 2, we have $2x + 4y = 0$ and $3y^2 + 2y + 4x = 0$. The first equation give $x = -2y$, which turns the second equation into $3y^2 - 6y = 0$. The solutions are $y = 0$ and $y = 2$. Since $x = -2y$, the critical points are $(0,0)$ and $(-4, 2)$.
   (b) We have $f_{xx} = 2$, $f_{xy} = 4$ and $f_{yy} = 6y + 2$. Thus $f_{xx} > 0$.
   At $(0,0)$, $f_{yy} = 2$ and $f_{xx}f_{yy} - (f_{xy})^2 < 0$ so the point is a saddle.
   At $(-4, 2)$, $f_{yy} = 14$ and $f_{xx}f_{yy} - (f_{xy})^2 > 0$ so the point is a (local) minimum.