1. Use the chain rule.
   (a) $g_s = g_x\, x_s + g_y\, y_s = g_x + g_y$.
   (b) Let $h = g_s$. Then

$$\begin{aligned} g_{ts} = g_{st} = h_t &= h_x\, x_t + h_y\, y_t = -h_x + 3h_y \\ &= -(g_{xx} + g_{xy}) + 3(g_{xy} + g_{yy}) = -g_{xx} + 2g_{xy} + 3g_{yy}. \end{aligned}$$

2. Note that $\nabla f = \langle 2x + 2y,\ 3y^2 + 8y + 2x\rangle$ and $\nabla f(0,1) = \langle 2, 11\rangle$.
   (a) $\mathbf{u} = \nabla f/|\nabla f| = 125^{-1/2}\langle 2, 11\rangle$.
   (b) The maximum is $|\nabla f| = \sqrt{125} = 5\sqrt{5}$.
   (c) We need $\mathbf{u} \cdot \nabla f = 0$.
   There are two possible answers: $125^{-1/2}\langle 11, -2\rangle$ and $125^{-1/2}\langle -11, 2\rangle$.

3. This could be done it at least two ways.
   - The tangent line is in a direction in which $D_{\mathbf{u}} = 0$. Such a vector was found in 2(c). Then $\langle x, y\rangle = t\mathbf{u} + \langle 0, 1\rangle$. Since all we need is a vector parallel to $\mathbf{u}$, we can drop the factor of $125^{-1/2}$ if we wish to get the cleaner formula
   $\langle x, y\rangle = t\langle 11, -2\rangle + \langle 0, 1\rangle$.
   - Since $dy/dx = -f_x/f_y$, we have $dy/dx = -2/11$. Thus the line is
   $y - 1 = (-2/11)(x - 0)$, which can be written $y = -2x/11 + 1$.

4. (a) We need $\nabla f = \mathbf{0}$. From Problem 2, this gives us $2x+2y = 0$ and $3y^2+8y+2x = 0$. The first equation give $x = -y$, which turns the second equation into $3y^2+6y = 0$. The solutions are $y = 0$ and $y = -2$. Since $x = -y$, the critical points are $(0, 0)$ and $(2, -2)$.
   (b) We have $f_{xx} = 2$, $f_{xy} = 2$ and $f_{yy} = 6y + 8$. Thus $f_{xx} > 0$.
   At $(0, 0)$, $f_{yy} = 8$ and $f_{xx}f_{yy} - (f_{xy})^2 > 0$ so the point is a (local) minimum.
   At $(2, -2)$, $f_{yy} = -4$ and $f_{xx}f_{yy} - (f_{xy})^2 < 0$ so the point is a saddle.