

1. $dx/dt = 4t + 3$ and $dy/dt = 3t^2 - 6t$.

(a) $\text{length} = \int_{-2}^4 \sqrt{(4t+3)^2 + (3t^2-6t)^2} dt$

(b) To be horizontal, $dy/dt = 0$, which means $t = 0$ or $t = 2$. This gives the two points $(-1, 2)$ and $(13, -2)$.

2. (a) Let $\mathbf{c} = \overrightarrow{BA} = \langle 1, 1, -1 \rangle$ and $\mathbf{a} = \overrightarrow{BC} = \langle x-1, 3, 3 \rangle$. Since \mathbf{a} must be perpendicular to \mathbf{c} , $\mathbf{a} \cdot \mathbf{c} = 0$. Thus $x = 1$ and so C is $(1, 3, 4)$ and $\mathbf{a} = \langle 0, 3, 3 \rangle$.

(b) Let $\mathbf{b} = \overrightarrow{AC} = \langle -1, 2, 4 \rangle$. The cosine is

$$\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{6 + 12}{\sqrt{9+9} \sqrt{1+4+16}} = \frac{18}{\sqrt{18} \sqrt{21}} = \sqrt{\frac{18}{21}} = \sqrt{\frac{6}{7}}.$$

3. (a) **First plane:** Since $\langle 1-0, 1-0, 0-0 \rangle$ and $\langle 1-0, 1-0, 2-0 \rangle$ are parallel to the plane, their cross product is a normal. The cross product is $\langle 2, -2, 0 \rangle$. Since the origin is in the plane, the equation is $2x - 2y = 0$ or, equivalently, $x - y = 0$.

Second plane: The equation is $\langle 1, 0, 2 \rangle \cdot \langle x-0, y-0, z-0 \rangle = 0$; that is, $x + 2z = 0$.

(b) We have the equations $x - y = 0$ and $x + 2z = 0$ for the planes. Since both must hold for the intersection, we could take, say, $z = t$. Then $x = -2t$ and $y = -2t$. In other words, $\langle x, y, z \rangle = t \langle -2, -2, 1 \rangle$. Of course, other answers are also valid, for example, $\langle x, y, z \rangle = t \langle 2, 2, -1 \rangle + \langle -2, -2, 1 \rangle$.

Alternatively, we could take the cross product of the two normals to the planes, $\langle 1, -1, 0 \rangle$ and $\langle 1, 0, 2 \rangle$ to get a vector in the direction of the line. We also need a point on the line. The origin works since the plane passes through the origin.

Alternatively, we could find two points on the line, P and Q . Then the line is $t\overrightarrow{PQ} + Q$. An obvious point is the origin. We can find another by choosing any nonzero value for x , y or z ; for example, with $x = 6$, $x - y = 0$ gives us $y = 6$ and $x + 2z = 0$ gives us $z = -3$, and so our point is $(6, 6, -3)$.