

1. $dx/dt = 6t^2 + 6t$ and $dy/dt = 2t - 3$.

(a) $\text{length} = \int_{-4}^2 \sqrt{(6t^2 + 6t)^2 + (2t - 3)^2} dt$

(b) To be vertical, $dx/dt = 0$, which means $t = 0$ or $t = -1$. This gives the two points $(-1, 2)$ and $(0, 6)$.

2. (a) Let $\mathbf{c} = \overrightarrow{BA} = \langle 1, 1, -1 \rangle$ and $\mathbf{b} = \overrightarrow{AC} = \langle x - 2, 2, 4 \rangle$. Since \mathbf{b} must be perpendicular to \mathbf{c} , $\mathbf{b} \cdot \mathbf{c} = 0$. Thus $x = 4$ and so C is $(4, 3, 4)$ and $\mathbf{b} = \langle 2, 2, 4 \rangle$.

(b) Let $\mathbf{a} = \overrightarrow{BC} = \langle 3, 3, 3 \rangle$. The cosine is

$$\frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}| |\mathbf{b}|} = \frac{6 + 6 + 12}{\sqrt{4 + 4 + 16} \sqrt{9 + 9 + 9}} = \frac{24}{\sqrt{24} \sqrt{27}} = \sqrt{\frac{24}{27}} = \frac{2\sqrt{2}}{3}.$$

3. (a) **First plane:** The equation is $\langle 1, 2, 0 \rangle \cdot \langle x - 0, y - 0, z - 0 \rangle = 0$; that is, $x + 2y = 0$.
Second plane: Since the line is in the plane, $\langle 1, 1, 0 \rangle$ is parallel to the plane. With $t = 0$, we see that $(0, 2, 0)$ is in the plane. (Any other value of t would work.) Since the origin is in the plane, $\langle 0, 2, 0 \rangle$ is parallel to the plane. Taking the cross product of the two vectors, we get the normal $\langle 0, 0, 2 \rangle$ and the equation of the plane is $2z = 0$ or, equivalently, $z = 0$.

(b) We have the equations $x + 2y = 0$ and $z = 0$ for the planes. Since both must hold for the intersection, we could take, say, $y = t$. Then $x = -2t$ and $z = 0$. In other words, $\langle x, y, z \rangle = t\langle -2, 1, 0 \rangle$. Of course, other answers are also valid, for example, $\langle x, y, z \rangle = t\langle 2, -1, 0 \rangle + \langle -4, 2, 0 \rangle$.

Alternatively, we could take the cross product of the two normals to the planes, $\langle 1, 2, 0 \rangle$ and $\langle 0, 0, 1 \rangle$ to get a vector in the direction of the line. We also need a point on the line. The origin works since the plane passes through the origin.

Alternatively, we could find two points on the line, P and Q and then the line would be $t\overrightarrow{PQ} + Q$. An obvious point is the origin. We can find another by choosing any nonzero value for x or y ; for example, with $x = 12$, $x + 2y = 0$ gives us $y = -6$. Since we have $z = 0$ from the second plane, our point is $(12, -6, 0)$.