YOU MUST SHOW YOUR WORK.

Q1. Evaluate the following. (a) $\displaystyle \frac{d}{dx}\int_1^{x^2} \sqrt{t^3+1}\,dt$ (b) $\displaystyle \int t\sqrt{t-1}\,dt$

Q2. Consider the region $\mathcal{R}$ bounded by the parabola $y = x(4-x) = 4x - x^2$ and the line $y = x$ for $0 \le x \le 3$.

(a) Find the area of the region $\mathcal{R}$.

(b) That part of the region $\mathcal{R}$ lying below $y = 3$ is rotated about the y-axis. Write down, but do not evaluate, an integral for the volume of the resulting region.

Q3. (a) Write out the form of the partial fraction decomposition, but do NOT determine the numerical values of the coefficients: $\displaystyle \frac{5}{(x^2-1)^2\,(x^2+1)}$.

(b) Evaluate $\displaystyle \int \frac{dx}{x^2\sqrt{x^2-1}}$.

(c) Evaluate $\displaystyle \int \frac{dx}{e^x+1}$.

Q4. (a) Write down the 3-interval Trapezoidal Rule approximation (T_3) to $\int_0^1 \sqrt{1-x^3}\,dx$. Don't do any simplification.

(b) Is the integral $\displaystyle \int_4^\infty \frac{dx}{(x-1)^2}$ convergent or divergent and why?

(c) Is the integral $\displaystyle \int_1^\infty \frac{(1+e^{-x})\,dx}{x}$ convergent or divergent and why?

(d) Some function $f(x)$ satisfies $f'(x) > 0$ and $f''(x) > 0$ for $0 \le x \le 4$. The values of $I = \int_0^4 f(x)\,dx$, R_{20} (20-interval Right Endpoint Rule) and T_{20} (20-interval Midpoint Rule) were computed. The three values were 5.02, 5.05 and 5.06. Which is I? R_{20}? T_{20}? No need to explain.

Q5. (a) For what values of b is $y = b^2x - 2b$ a solution of $(xy' - y)^2 = 4y'$? (There is at least one value.)

(b) For what values of c is $y = c/x$ a solution of $(xy' - y)^2 = 4y'$? (There is at least one value.)

(c) The curve given by $x^2y^3 = 64$ that lies between the points $(1, 4)$ and $(8, 1)$ is rotated about the y-axis. Write down an integral for the area of the surface? For full credit, do arithmetic. For example write 4, not $64^{1/3}$, and 144, not 12^2.

Q6. (a) Write $\sin x \cos(2x)$ as a sum of exponential functions.
Your answer should contain complex numbers.

(b) Expand $\dfrac{2x}{x^2+4}$ in partial fractions so that the denominators are all first degree in x.

(c) Find real numbers x and y so that $e^{x+iy} = -1$.
You may NOT leave logarithms or trig functions in your answer.