1. (a) Use the substitution $x = u^2$ so that $dx = 2u\,du$. Then

$$\int_0^9 \frac{dx}{1+\sqrt{x}} = \int_0^3 \frac{2u\,du}{1+u} = 2\int_0^3 \left(1 - \frac{1}{1+u}\right) du$$
$$= 2(u - \ln|1+u|)\Big]_0^3 = 2(3 - \ln 4).$$

(b) This can be done using the identity $\cos^2 x = \frac{\cos(2x)+1}{2}$ and integrating by parts. It can also be done with complex numbers, which is what is done here.

$$\int \cos^2 x\, e^x\, dx = \int \left(\frac{e^{ix}+e^{-ix}}{2}\right)^2 e^x\, dx$$
$$= \frac{1}{4}\int (e^{(1+2i)x} + 2e^x + e^{(1-2i)x})\, dx$$
$$= \frac{e^{(1+2i)x}}{4(1+2i)} + \frac{e^x}{2} + \frac{e^{(1-2i)x}}{4(1-2i)} + C.$$

Since you can leave complex numbers in your answer, you can stop here.

(c) You can use trig substitution, but it is much easier to set $1 - x^2 = t$ so that $-2x\,dx = dt$ and then we have

$$\int x\sqrt{1-x^2}\, dx = \frac{-1}{2}\int \sqrt{t}\, dt = \frac{-1}{2}\frac{t^{3/2}}{3/2} + C = \frac{-(1-x^2)^{3/2}}{3} + C.$$

(d) This is an improper integral because we are dividing by zero when $x = 0$. Thus we need to write $\int_{-1}^1 = \int_{-1}^0 + \int_0^1$. It turns out that neither of the integrals on the right converges and so the original integral diverges. To see the divergence of $\int_0^1$:

$$\int_0^1 \frac{dx}{x^2} = \lim_{a\to 0^+} \int_a^1 x^{-2} dx = \lim_{a\to 0^+} (a^{-1} - 1) = \infty.$$

Alternatively, you can use a test discussed in class:

$$\int_0^b x^{-p} dx \text{ converges if and only if } p < 1.$$

2. (a) The values of $A$ are given by $y' = 0$. Thus $A = -2$, $0$ and $3$.

(b) The answer is 3. When $y > 3$, $y' < 0$ Thus $y$ decreases toward $A = 3$.

(c) The answer is $-2$. When $-2 < y < 0$, $y' < 0$. Thus $y$ decreases toward $A = -2$.

3. $\mathcal{R}$ looks like a right triangle with vertices at (0,0), (4,0) and (4,2) and the hypotenuse bulging up.

   (a) $\int_0^4 \sqrt{x}\,dx$ or $\int_0^2 (4-y^2)\,dy$.

   (b) $\int_0^2 \pi(4^2-y^4)\,dy$ or $\int_0^4 2\pi x\sqrt{x}\,dx$, if you studied the method in Section 6.3 on your own.

   (c) The answer has the form $\int 2\pi x\sqrt{(dx)^2+(dy)^2}$, which needs to be converted to an integral over either $x$ or $y$:

$$2\pi \int_0^4 x\sqrt{1+1/\big(2\sqrt{x}\big)^2}\,dx \qquad \text{or} \qquad 2\pi \int_0^2 y^2\sqrt{(2y)^2+1}\,dy.$$

4. Separating variables: $e^x y' = -1$ and so $dy = -e^{-x}dx$. Integrating: $y = e^{-x} + C$.

5. We get two loops because the curve passes through the origin when $1 - 2\cos\theta = 0$. This happens when $\theta = \cos^{-1}(1/2) = \pm\pi/3$ (which is $\pm 60°$). The inner loop occurs when $|r|$ is smaller. Thinking about that, or graphing $r$ versus $\theta$ in polar coordinates, or graphing $r$ versus $\theta$ in Cartesian coordinates, we can see that this happens when $-\pi/3 \le \theta \le \pi/3$. Thus the answer is $\int_{-\pi/3}^{\pi/3} \frac{(1-2\cos\theta)^2}{2}\,d\theta$. You could also have done half a loop ($0 \le \theta \le \pi/3$) and doubled the integral.

6. (a) is an ellipse, (b) and (d) are hyperbolas and (c) is degenerate because no values of $x$ and $y$ satisfy the equation.

7. (a) $\frac{1+i}{2+i} = \frac{1+i}{2+i}\frac{2-i}{2-i} = \frac{(1+i)(2-i)}{2^2+1^2} = (3/5) + (1/5)i$.

   (b) $e\cos 2 + (e\sin 2)i$.

8. Since $\arg(i) = \pi/2$, the roots have arguments $\frac{2k\pi+\pi/2}{2004}$. To get the value closest to $-1$, we want the value closest to $\arg(-1) = \pi$. This occurs when $k = 1002$ because we then have $\pi + \pi/4008$. The answer is $\pi + \pi/4008$.