- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  One page of notes (both sides) is allowed.
- **You must show your work to receive credit.**

1. (24 pts.) For each of the following integrals, either evaluate the integral or prove that it diverges. You may leave complex numbers in your answers.

   (a) $\displaystyle\int_0^9 \frac{dx}{1+\sqrt{x}}$ (b) $\displaystyle\int \cos^2 x\, e^x\, dx$ (c) $\displaystyle\int x\sqrt{1-x^2}\, dx$ (d) $\displaystyle\int_{-1}^1 \frac{dx}{x^2}$

2. (9 pts.) Consider the differential equation $dy/dt = y(2+y)(3-y)$.

   (a) Find all constants A such that $y(t) = A$ is a solution to the differential equation. You must briefly explain how you determined the values.

   (b) If the initial condition is $y(0) = 4$, what is $\lim_{t\to\infty} y(t)$? You must briefly explain why.

   (c) If the initial condition is $y(0) = -1$, what is $\lim_{t\to\infty} y(t)$? You must briefly explain why.

   Note: An answer to (b) or (c) might be ∞ or $-\infty$.

3. (18 pts.) Let $\mathcal{R}$ be the region bounded above by the curve $x = y^2$, on the right by the line $x = 4$, and below by the x-axis. In this problem, do NOT evaluate the integrals.

   (a) Sketch $\mathcal{R}$ and write down an integral for its area.

   (b) The region $\mathcal{R}$ is rotated about the y-axis, giving a solid $\mathcal{V}$. Write down an integral for the volume of $\mathcal{V}$.

   (c) The inner surface of $\mathcal{V}$ is generated by rotating the part of the curve $x = y^2$ that is on the boundary of $\mathcal{R}$. Write down an integral for the area of the inner surface of $\mathcal{V}$.

4. (5 pts.) Find the general solution to the differential equation $e^x y' + 1 = 0$.

5. (6 pts.) The equation $r = 1 - 2\cos\theta$ describes a curve in polar coordinates that looks like a loop within a loop. Write down an integral for the area of the inner loop.

6. (9 pts.) Identify each of the following conic sections as an ellipse (includes circle), hyperbola, parabola or degenerate (no curve or intersecting straight lines).

   (a) $x^2 + 3y^2 - 1 = 0$ (b) $x^2 - 3y^2 - 1 = 0$

   (c) $x^2 + 3y^2 + 1 = 0$ (d) $x^2 - 3y^2 + 1 = 0$

7. (6 pts.) Express each of the following as $x + yi$ where x and y are real numbers. You may leave trig functions in your answer.

   (a) $\dfrac{1+i}{2+i}$ (b) e^{1+2i}

8. (3 pts.) The complex number $i^{1/2004}$ has 2004 values. Let z be the value that is closest to -1. What is $\arg(z)$?

   "Closest to -1" means the distance between -1 and z when they are plotted in the plane is as small as possible. In other words, $|z-(-1)| = |z+1|$ is as small as possible.

   Hint: Look at it geometrically. The values of i^{2004} are points on the circle of radius 1 centered at the origin.

END OF EXAM