Math 20B Second Midterm Solutions November 15, 2004

1. The curve goes from $x = 0$ to $x = 1$. Solving for $y(x)$, we have $y = \sqrt{1 - x^4}$. Thus $y' = -2x^3(1 - x^4)^{-1/2}$.

(a) $$\int_0^1 \sqrt{1 + (y')^2}\, dx = \int_0^1 \sqrt{1 + \frac{4x^6}{1 - x^4}}\, dx = \int_0^1 \sqrt{\frac{1 - x^4 + 4x^6}{1 - x^4}}\, dx.$$

(b) $$\begin{aligned}\int_0^1 2\pi y(x)\sqrt{1 + (y')^2}\, dx &= 2\pi \int_0^1 \sqrt{(1 - x^4)\left(1 + \frac{4x^6}{1 - x^4}\right)}\, dx \\ &= 2\pi \int_0^1 \sqrt{1 - x^4 + 4x^6}\, dx.\end{aligned}$$

2. (a) $0.08/4^2 = 0.005$, since error is roughly C/n^2 and we have multiplied n by 4.

   (b) The error in the Midpoint Rule is about half that of the Trapezoidal Rule and opposite in sign. Hence the answer is -0.04. (You will get partial credit for 0.04.)

3. (a) $2\cos(3\pi/4) + 2i\sin(3\pi/4) = -\sqrt{2} + \sqrt{2}\, i.$

   (b) $r = 2^{1/3}$. The values of θ are

$$\frac{3\pi/4}{3} = \frac{\pi}{4}, \qquad \frac{3\pi/4 + 2\pi}{3} = \frac{11\pi}{12}, \qquad \frac{3\pi/4 + 4\pi}{3} = \frac{19\pi}{12}.$$

(You need not do the arithmetic to simplify the angles.)

4. Separating variables: $\frac{dy}{y} = 2x\, dx$ and so $\ln y = x^2 + C$. Using $y(0) = 2$, we have $\ln 2 = 0^2 + C$ and so $C = \ln 2$. At $x = 3$ we have $\ln y = 3^2 + C = 9 + \ln 2$ and so $y = e^{9+\ln 2} = 2e^9$.

5. We have

$$\frac{2x^2}{x^2 - 1} = 2 + \frac{2}{(x - 1)(x + 1)} = 2 + \frac{1}{x - 1} - \frac{1}{x + 1},$$

where it is up to you how you find the partial fractions. Integrating gives $2x + \ln\left(\frac{x-1}{x+1}\right)$+C.