

1. The curve goes from  $x = 0$  to  $x = 1$ . Solving for  $y(x)$ , we have  $y = \sqrt{1 - x^4}$ . Thus  $y' = -2x^3(1 - x^4)^{-1/2}$ .

$$(a) \int_0^1 \sqrt{1 + (y')^2} dx = \int_0^1 \sqrt{1 + \frac{4x^6}{1 - x^4}} dx = \int_0^1 \sqrt{\frac{1 - x^4 + 4x^6}{1 - x^4}} dx.$$

$$(b) \int_0^1 2\pi y(x) \sqrt{1 + (y')^2} dx = 2\pi \int_0^1 \sqrt{(1 - x^4) \left(1 + \frac{4x^6}{1 - x^4}\right)} dx \\ = 2\pi \int_0^1 \sqrt{1 - x^4 + 4x^6} dx.$$

2. (a)  $0.08/4^2 = 0.005$ , since error is roughly  $C/n^2$  and we have multiplied  $n$  by 4.  
 (b) The error in the Midpoint Rule is about half that of the Trapezoidal Rule and opposite in sign. Hence the answer is  $-0.04$ . (You will get partial credit for 0.04.)
3. (a)  $2 \cos(3\pi/4) + 2i \sin(3\pi/4) = -\sqrt{2} + \sqrt{2}i$ .  
 (b)  $r = 2^{1/3}$ . The values of  $\theta$  are

$$\frac{3\pi/4}{3} = \frac{\pi}{4}, \quad \frac{3\pi/4 + 2\pi}{3} = \frac{11\pi}{12}, \quad \frac{3\pi/4 + 4\pi}{3} = \frac{19\pi}{12}.$$

(You need not do the arithmetic to simplify the angles.)

4. Separating variables:  $\frac{dy}{y} = 2x dx$  and so  $\ln y = x^2 + C$ . Using  $y(0) = 2$ , we have  $\ln 2 = 0^2 + C$  and so  $C = \ln 2$ . At  $x = 3$  we have  $\ln y = 3^2 + C = 9 + \ln 2$  and so  $y = e^{9 + \ln 2} = 2e^9$ .

5. We have

$$\frac{2x^2}{x^2 - 1} = 2 + \frac{2}{(x - 1)(x + 1)} = 2 + \frac{1}{x - 1} - \frac{1}{x + 1},$$

where it is up to you how you find the partial fractions. Integrating gives  $2x + \ln\left(\frac{x-1}{x+1}\right) + C$ .