1. (a)
$$\frac{d}{dx}\int_{-x}^{2x}\sqrt{u^3+1}\,du = \frac{d}{dx}\int_0^{2x}\sqrt{u^3+1}\,du - \frac{d}{dx}\int_0^{-x}\sqrt{u^3+1}\,du$$
$$= 2\sqrt{8x^3+1}+\sqrt{1-x^3}.$$

(b) Integrate by parts with $u=\sin^{-1}x$ and $dv=dx$ and then let $1-x^2=t$:
$$\sin^{-1}x\,dx = x\sin^{-1}x-\int\frac{x\,dx}{\sqrt{1-x^2}} = x\sin^{-1}x+\frac{1}{2}\int t^{-1/2}\,dt$$
$$= x\sin^{-1}x+t^{1/2}+C = x\sin^{-1}x+\sqrt{1-x^2}+C.$$

(c)
$$\int_0^{\pi/2}\cos^3 x\,dx = \int_0^{\pi/2}(\cos x-\sin^2 x\cos x)\,dx = \left(\sin x-\frac{\sin^3 x}{3}\right]_0^{\pi/2} = \frac{2}{3}.$$

(d) Use substitution with $t=3x-1$:
$$\int_{x=0}^{x=1}(3x-1)^4\,dx = \int_{t=-1}^{t=2}\frac{t^4\,dt}{3} = \frac{t^5}{3\times 5}\Big]_{-1}^{2} = \frac{2^5+1}{3\times 5} = \frac{11}{5}.$$

You can leave your answer as, e.g., $\frac{2^5+1}{3\times 5}$.

2. The easiest way to prove it is to use the Fundamental Theorem of Calculus: Verify that the derivative of $x\sin(\ln x)+C$ is the integrand.

3. The curve intersects the x-axis at $x=-2$, $x=0$ and $x=1$. The function is positive for $-2<x<0$ and negative for $0<x<1$. The area is
$$\int_{-2}^{1}|x^3+x^2-2x|\,dx = \int_{-2}^{0}(x^3+x^2-2x)\,dx+\int_0^1-(x^3+x^2-2x)\,dx$$
$$= \left(\frac{x^4}{4}+\frac{x^3}{3}-x^2\right]_{-2}^{0} - \left(\frac{x^4}{4}+\frac{x^3}{3}-x^2\right]_0^1$$
$$= -\left(\frac{16}{4}-\frac{8}{3}-4\right)-\left(\frac{1}{4}+\frac{1}{3}-1\right)$$
$$= \frac{7}{3}-\frac{1}{4}+1 = \frac{37}{12}.$$

4. The curves intersect at $(0,0)$ and $(1,1)$. Thus the integral is
$$\pi\int_0^1\left((x^{1/2}+2)^2-(x^2+2)^2\right)\,dx.$$