Each quiz is worth 12 points.

Q1. These integrals could be done using substitution; however, substitution is not covered on this quiz; therefore, the solutions are done without it. If you did use substitution, you will still get credit.

$$\int_0^8 \sqrt{\frac{2}{t}}\,dt \;=\; \int_0^8 \sqrt{2}\,t^{-1/2}\,dt \;=\; \sqrt{2}\,\frac{t^{1/2}}{1/2}\bigg|_0^8 \;=\; 8.$$

At some point you must write down an antiderivative of something like at^n. If you *do not* get that correct, you receive no credit.
If you manipulate the integrand incorrectly to start with (e.g. convert it to $\sqrt{2}\,t^{1/2}$) but integrate what you get correctly, you get 3 points. If you get to all but the 8 at the end, you get 5 points.

Due to poor design of the problem the integrand, $\sqrt{2/t}$, is not defined at $t = 0$. Therefore, anyone noting this fact and saying the integral cannot be done will receive credit for the problem.

$$\int e^{x+1}\,dx \;=\; \int e\,e^x\,dx \;=\; e\,e^x + C \;=\; e^{x+1} + C.$$

Getting e^{x+1} (or $e\,e^x$) is worth 4 points. Having the C (even if you don't have e^{x+1}) is worth 2 points.

Q2 Each part is 4 points.

(a) Use the substitution $u = \ln t$ to get $\int u\,du \;=\; u^2/2 + C \;=\; (\ln t)^2/2 + C$.
Lose 2 points for leaving answer as $u^2/2 + C$. Lose 1 point for omitting C. If you get the wrong answer for some reason but have $+C$, get 1 point.

(b) Use the substitution $2x - 3 = u$ to get $\int (u+3)u^{50}\,du$, possibly with limits. The indefinite integal is $u^{52}/52 + 3u^{51}/51 + C$. There are three approaches:
(i) Evaluate the indefinite integral, getting $(2x-3)^{52}/52 + (2x-3)^{51}/17 + C$ and substitute in the limits.
(ii) Carry along the limits as $x = 1$ and $x = 2$, evaluate the integral as before (no $+C$ needed) and then substitute.
(iii) Change the limits to values of u:

$$\int_1^2 4x(2x-3)^{50}\,dx \;=\; \int_{x=1}^{x=2} (u+3)u^{50}\,du \;=\; \int_{u=-1}^{u=1} (u+3)u^{50}\,du$$
$$=\; \Big(u^{52}/52 + 3u^{51}/51\Big|_{-1}^{1} \;=\; 2/17.$$

If you don't substitute inside the integral for all the x values, including dx, when changing variables, no credit. If you substitute the values $x = 1, 2$ for u instead of $u = -1, 1$ when evaluating the definite integral, lose 3 points. If you use a correct approach but make an algebra error, lose 1 point.

(c) The first two curves intersect at $x = 0$, the answer is $\int_0^2 (e^x - (x+1))\,dx$. Either of the answers $\left|\int_0^2 (e^x - (x+1))\,dx\right|$ or $\int_0^2 |e^x - (x+1)|\,dx$ is also acceptable.
If you get the integrand wrong, lose 2 points. Each limit on the integral is worth 1 point.

Q3. (a) Integrate by parts with $u = \ln x$ and $dv = x^{-2}dx$:

$$\int \frac{\ln x}{x^2}\,dx \;=\; \frac{-\ln x}{x} + \int \frac{dx}{x^2} \;=\; \frac{-\ln x}{x} - \frac{1}{x} + C.$$

1 point for right choice of u and dv, 1 point for $+C$, 2 points for rest of it.

Use integration by parts with $u = x$ and $dv = \cos x\,dx$:

$$\int_0^\pi x\cos x\,dx \;=\; x\sin x\Big|_0^\pi - \int_0^\pi \sin x\,dx \;=\; \cos x\Big|_0^\pi \;=\; -2.$$

1 point for right choice of u and dv, 2 points for carrying out the indefinite integration, 1 point for correct evaluation at 0 and π.

(b) $\pi \displaystyle\int_{-1}^{1} \left((1-(-2))^2 - (x^4 - (-2))^2\right)dx$, or some equivalent rewrite of it.
If you used method of cylinders (Sec. 6.3; not assigned):

$$2\pi \int_0^1 (y+2)(y^{1/4} - (-y^{1/4}))\,dy.$$

Q4. #1. The first integral must be written as a sum of two because of the problem and $x = 0$. There is one point for splitting the integral. If you try to do (a) without splitting the integral, there is no credit.
Since $\int x^{-2}dx \;=\; -1/x + C$, we have

$$\int_0^1 \frac{dx}{x^2} \;=\; \lim_{a\to 0^+}\left(\frac{1}{a} - 1\right) \;=\; \infty \qquad \text{and} \qquad \int_1^\infty \frac{dx}{x^2} \;=\; \lim_{b\to\infty}\left(1 - \frac{1}{b}\right) \;=\; 1.$$

Hence the first integral diverges and the second converges.
Lose 1 point total if you fail to write either or both integrals as limits and simply say something like

$$\int_1^\infty \frac{dx}{x^2} \;=\; \frac{-1}{x}\Big|_1^\infty \;=\; 0 - (-1) \;=\; 1.$$

There is another way to do these integrals: The second integral can be done by the result in the text for $\int_1^\infty \frac{1}{x^p}dx$, provided you use it correctly. You can also use the result for $\int_0^b \frac{1}{x^p}dx$ discussed in class. Using these correctly gives full credit. Using them incorrectly gives no credit.

#2. Since you need not do the arithmetic, if you use the right formulas and right numbers and then do the arithmetic incorrectly, you lose no points.

(a) The text gives the error bound $K(b-a)^3/12n^2$, where K is a bound on the second derivative. Thus we take $K = 36$, $a = -1$, $b = 3$ and $n = 8$ to obtain $\frac{36\times 4^3}{12\times 8^2}$. You can leave the answer like this or simplify it to 3.

(b) In the supplement's notation, an estimate for the error is $C_T/8^2$ and, by (3),$C_T \approx \frac{4\times 4^2(5-8)}{3}$. You can leave it like this, or you can simplify it to obtain -1 for the estimated error.
Getting C_T is worth 2 points. Knowing you want $C_T/8^2$ is worth 1 point.

Incidentally, I was integrating $2x^3 - 9$ and the value of the integral is 4.

Q5. (a) Since $x_0 = 0$ and $h = 0.5$, we want y_2. We have $y_0 = y(0) = 1$,

$$y_1 = y_0 + F(x_0, y_0) = 1 + 0.5 \times (2 \times 1 + 4 \times 0) = 2,$$

$$y_2 = y_1 + hF(x_1, y_1) = 2 + 0.5 \times (2 \times 2 + 4 \times 0.5) = 5.$$

If you know the formula for Euler's method and how to use it, 2 points.
If you use the formula and correctly get y_1, 1 more point.

(b) We have $y' = 2x + A$ and $xy' - 2y = x(2x + A) - 2(x^2 + Ax) = -Ax$. Thus $A = -3$.
If your work shows you need to compute y' and substitute it into the differential equation to find A, but you did not get $A = -3$, lose one point.

(c) You have to solve for x or y and then use the formula for arc length. Also, you will need to notice that the values of x (or y) start at zero (since we are in the first quadrant) and go to 1. Solving for y:

$$y = (1 - x^2)^{1/4}, \quad y' = -\frac{x}{2}(1 - x^2)^{-3/4}, \quad \text{length} = \int_0^1 \sqrt{1 + \frac{x^2}{4(1 - x^2)^{3/2}}}\, dx.$$

If you solved for x instead:

$$x = (1 - y^4)^{1/2}, \quad x' = -2y^3(1 - y^4)^{-1/2}, \quad \text{length} = \int_0^1 \sqrt{1 + \frac{4y^6}{1 - y^4}}\, dy.$$

Partial credit: Knowing the formula for arc length as indicated by your work, 1 point.
Knowing you need to solve for x or y as indicated by your work, 1 point.
Also getting the correct value for x' or y', 1 more point.
Getting the correct limits on the integral, 1 point.

Q6. 1.(a) $\dfrac{10}{2+i} = \dfrac{10}{2+i}\dfrac{2-i}{2-i} = \dfrac{10(2-i)}{2^2+1^2} = 2(2-i) = 4 - 2i. \quad (\text{or } 4 + (-2)i)$

(b) $e^{(1+i)\pi} = e^{\pi+i\pi} = e^{\pi}\cos\pi + ie^{\pi}\sin\pi = -e^{\pi}. \quad (\text{or } -e^{\pi} + 0i)$

2. (a) Multiply out: $x^2 - y^2 = 4$, a hyperbola (b) a parabola