YOU MUST SHOW YOUR WORK.

Q1. Evaluate the following integrals. $\displaystyle\int_0^8 \sqrt{\frac{2}{t}}\,dt$ $\displaystyle\int e^{x+1}\,dx$

Q2. (a) Evaluate $\displaystyle\int \frac{\ln t}{t}\,dt$.

(b) Evaluate $\int_1^2 4x(2x-3)^{50}\,dx$.

(c) Write down an integral for the area of the region enclosed by the three curves

$$y = e^x, \qquad y = x+1, \qquad x = 2.$$

Q3 (a) (8 pts) Evaluate the integrals

$$\int \frac{\ln x}{x^2}\,dx \qquad \int_0^\pi x\cos x\,dx \text{ (no trig functions in answer).}$$

(b) (4 pts) Set up, but **do not evaluate** an integral for the volume obtained by rotating the region between $y = x^4$ and $y = 1$ about the line $y = -2$.

Q4 #1 (6 pts) Determine if the following integrals converge or diverge. Remember to give a reason for your answer.

$$\text{(a)} \quad \int_{-1}^1 \frac{dx}{x^2} \qquad \text{(b)} \quad \int_1^\infty \frac{dx}{x^2}.$$

Q4 #2 (6 pts) Estimating $\int_{-1}^3 f(x)\,dx$ using the Trapezoidal Rule, I obtained $T_4 = 8$ and $T_8 = 5$. I also know that $|f'(x)| \le 54$ and $|f''(x)| \le 36$ for $-1 \le x \le 3$.

(a) Find a guaranteed bound on the error in T_8.

(b) Find a reasonable estimate for the error in T_8.

Q5. (a) (4 pts) Use Euler's method with step size $h = 0.5$ to estimate $y(1)$ where

$$y'(x) = 2y + 4x \qquad \text{and} \qquad y(0) = 1.$$

Do the arithmetic!

(b) (4 pts) Find a value of A so that $y = x^2 + Ax$ is a solution to the differential equation $x(dy/dx) - 2y = 3x$.

(c) (4 pts) Set up, but do not evaluate, an integral for the length in the first quadrant of the curve $x^2 + y^4 = 1$.

Q6. 1. Express the following as $a + bi$, where a and b are real numbers and do NOT contain trig functions.

$$\text{(a)} \quad \frac{10}{2+i} \qquad \text{(b)} \quad e^{(1+i)\pi}.$$

2. In each case, indicate if the curve is an ellipse, hyperbola or parabola.

$$\text{(a)} \quad x + y = \frac{4}{x-y} \qquad \text{(b)} \quad r = \frac{3}{1-\sin\theta}.$$