- Print Name, ID number and Section on your blue book.
- BOOKS and CALCULATORS are NOT allowed.
  Two sheets of NOTES are allowed.
- **You must show your work to receive credit.**

1. (36 pts.) For each of the following integrals, either evaluate the integral or prove that it diverges.

   (a) $\displaystyle\int_1^e x^3 \ln x \, dx$ (b) $\displaystyle\int_0^{\pi/2} \frac{2+\cos x}{x}\, dx$ (c) $\displaystyle\int (1+2u)^{10}\, du$

   (d) $\displaystyle\int_0^{1/\sqrt{2}} \frac{x^2}{\sqrt{1-x^2}}\, dx$ (e) $\displaystyle\int_1^2 \frac{1}{u+u^2}\, du$ (f) $\displaystyle\int e^{t+e^t}\, dt$

2. (9 pts.) Express the following in the Cartesian form $a+bi$ or the polar form (r,θ), as indicated.

   Do not leave trig functions in your answers.

   (a) The values of $w = \dfrac{1+i}{1-3i}$ and $\overline{w}$ in Cartesian form.

   (b) The values of $(1-i)^{1/3}$ in polar form.

   (c) The value of $e^{2+\pi i/4}$ in Cartesian form.

3. (15 pts.) The equation $x^4 + y^4 = 1$ describes a curve that looks somewhat like a square with rounded corners. Set up integrals for each of the following.

   Do not evaluate the integrals.

   (a) The area of the region enclosed by the curve.

   (b) The perimeter of the region; that is, the length of the curve that encloses the region.

   (c) The volume of the region obtained when the curve is rotated about the line $y=-2$.

   Remark: The equation can be solved giving two functions $y(x)$, namely
   $y = (1-x^4)^{1/4}$ and $y = -(1-x^4)^{1/4}$.

4. (9 pts.) Identify each of the following as either an ellipse, hyperbola or parabola.

   (a) $y^2 = 3+2x^2$ (b) $r = \dfrac{2}{2+\cos\theta}$ (c) $y^2 = 3-2x^2$

5. (12 pts.) Find the indicated solutions of the following differential equations.
   (a) The particular solution of $dx/dt = x^2$, with the intial condition $x(0) = 0$.
   (b) The general solution of $xy' + y = 1$.

6. (5 pts.) Set up an integral for the area that lies inside the polar curve $r = 1 + \cos\theta$ and outside the polar curve $r = 1$.

   Do not evaluate the integral.

7. (10 pts.) Let $x = \tan(t/2)$.
   (a) Express dt/dx as a rational function of x.

   Using the complex forms

$$\sin(t/2) = \frac{e^{it/2} - e^{-it/2}}{2i} \quad \text{and} \quad \cos(t/2) = \frac{e^{it/2} + e^{-it/2}}{2},$$

   we have $x = \frac{e^{it/2} - e^{-it/2}}{i(e^{it/2} + e^{-it/2})}$. With some algebraic manipulation, this can be converted to

$$e^{it} = \frac{1 + ix}{1 - ix}. \tag{1}$$

   (b) Using (1), express $\sin t$ and $\cos t$ as rational functions of x with no imaginary numbers present.

   This is called the *half angle substitution*. By using it, an integrand that is a rational function of $\sin t$ and $\cos t$ can be converted into a rational function of x. The integrand can then be evaluated by using partial fractions.

8. (4 pts.) The left, right, Trapezoidal and Midpoint rules with $n = 100$ were used to estimate $I = \int_0^1 e^{-x^2/2}\,dx$. Calling the estimates L_{100}, R_{100}, T_{100} and M_{100}, arrange them and I in order starting with the smallest and ending with the largest. For example, someone might get the arrangement $L_{100} < R_{100} < T_{100} < I < M_{100}$ — but don't copy this since it is not correct.

   You must correctly explain how you obtained your arrangement.

END OF EXAM