1. (a) Integrate by parts with $u = x$, $dv = e^{2x}dx$:

$$\int xe^{2x}\,dx \;=\; \frac{xe^{2x}}{2} - \int \frac{e^{2x}}{2}\,dx \;=\; \frac{xe^{2x}}{2} - \frac{e^{2x}}{4} + C.$$

(b) Easiest is the substitution $x^2 + 9 = u$:

$$\int_0^4 \frac{x}{\sqrt{9+x^2}}\,dx \;=\; \int_{x=0}^{x=4} \frac{u^{-1/2}}{2}\,du \;=\; u^{1/2}\Big|_9^{2}5 \;=\; 5-3 \;=\; 2.$$

(c) Easiest is the substitution $u = e^{-x}$, but $u = e^x$ or $u = e^x + 1$ is more natural. Using $u = e^x$, $du = e^x\,dx$:

$$\int \frac{1}{e^x+1}\,dx \;=\; \int \frac{1}{u+1}\frac{du}{e^x} \;=\; \int \frac{1}{u(u+1)}\,du \;=\; \int \left(\frac{1}{u} - \frac{1}{u+1}\right)du$$
$$=\; \ln\left(\frac{u}{u+1}\right) + C \;=\; \ln\left(\frac{e^x}{e^x+1}\right) + C \;=\; -\ln(1+e^{-x}) + C.$$

(d) Since the function $f(t) = \sin(t^3)$ is odd (that is, $f(-t) = -f(t)$), the integral is zero.

(e) By the Fundamental Theorem of Calculus $g(x) = F(2003) - F(x^2)$ where $F(t)$ is an antiderivative of $f(t) = \sin(t^3)$. Thus

$$g'(x) \;=\; -F(x^2)(x^2)' \;=\; -f(x^2)(2x) \;=\; -2x\sin(x^6).$$

Alternatively, $g(x) = -\int_{2003}^{x^2} \sin(t^3)\,dt$, so $g'(x) = -\sin((x^2)^3)\,(x^2)' = -2x\sin(x^6)$.

2. (a) The parabola is positive for $-3 < x < 3$, so the average is $\frac{1}{6}\int_{-3}^{3}(9-x^2)\,dx$.

(b) The curves intersect at $(0,0)$ and $(1,1)$. The volume is

$$\pi\int_0^1 ((x^{1/2})^2 - x^2)\,dx \;=\; \pi\int_0^1 (x - x^2)\,dx.$$

If you used the method of cylinders, which we did not discuss, you would get $2\pi\int_0^1 (x-x^2)x\,dx$.

END OF EXAM