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Signature _____ ID # _____

- Use this exam for your work. **Do NOT use a blue book**.
- Use the back of the exam pages if you need scratch paper.
- The exam is CLOSED BOOK, but you may use ONE SHEET OF NOTES.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (40 pts.) Let $\mathcal{R}$ be the region between the two parabolas $y = x^2$ and $x = y^2$. Let $\mathcal{V}$ be the volume obtained when $\mathcal{R}$ is rotated about the y-axis.

   (a) Sketch the region $\mathcal{R}$. Include in your drawing the coordinates of the point where the parabolas intersect.

Set up (but *do not evaluate*) integrals for the following.

(b) The arc length of the boundary of $\mathcal{R}$ that is on the parabola $y = x^2$.

(c) The volume of $\mathcal{V}$.

(d) The surface area of $\mathcal{V}$. *Be careful*: $\mathcal{V}$ has what might be called an inner and outer surface. The surface area is the sum of the areas of these two surfaces.

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2. (20 pts.) Given the two curves $r = 2$ and $r = 4\cos\theta$ in polar coordinates.

   (a) Find the polar coordinates of the points where the curves intersect.

   (b) Set up (but *do not evaluate*) an integral for the area that lies inside the curve $r = 4\cos\theta$ but outside the curve $r = 2$; that is, the area of the region for which $2 \le r \le 4\cos\theta$.

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3. (30 pts.) Express each of the following in the form $a + bi$ with a and b real numbers.

   Do **NOT** leave trig functions in your answers.

   (a) $(2 + 4i)/(1 - 7i)$

   (b) $\left(2\sqrt{3} + 2i\right)^{20}$

   (c) $e^{3+i\pi/2}$

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4. (20 pts.) (a) Determine whether $\int_0^\infty e^{-x}dx$ is convergent or divergent.

(b) Use part (a) and the comparison theorem to determine whether $\int_0^\infty \frac{e^{-x}}{2+\sin x}\,dx$ is convergent or divergent.

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5. (30 pts.) Evaluate the following integrals.

(a) $\int \frac{\ln x}{x^2}\,dx$

(b) $\int \cos x\ \cos(3x)\,dx$

(c) $\int \frac{dx}{x^2\sqrt{1-x^2}}$

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6. (10 pts.) Write out the partial fraction decomposition of the function $\dfrac{x}{x^2-1}$.

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7. (15 pts.) Solve the differential equation $\dfrac{dy}{dx} = \dfrac{x^2+1}{x^2 y}$ with the initial condition $y(1) = -2$ for y as a *function* of x; that is, find $y(x)$.

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8. (15 pts.) Use Euler's method with step size 1.0 to estimate $x(3.0)$, where $x(t)$ is the solution of the initial value problem

$$\frac{dx}{dt} = x + t, \quad x(0) = 0.$$

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9. (20 pts.) Consider the integral $I = \int_0^2 \frac{2}{4x+1}\, dx$.

(a) Use the Midpoint rule with $n = 2$ subintervals to approximate I.

(b) How large should n be so that the midpoint approximation of I is accurate to within 6×10^{-4}?