Math 20B Second Exam (125 points) 4:40 PM March 5, 2001

- Please put your name, ID number, and section number (or time) on your blue book.
- The exam is CLOSED BOOK, but you may use ONE SHEET OF NOTES.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (80 pts.) Evaluate the following integrals. Remember to show your work!

   (a) $\displaystyle\int_0^{1/2} \frac{x}{\sqrt{1-x^2}}\,dx$ (b) $\displaystyle\int \sqrt{t}\left(t+\sqrt[3]{t}\right)\,dt$

   (c) $\displaystyle\int \sin^2 t\,\cos^3 t\,dt$ (d) $\displaystyle\int (\ln x)^2\,dx$

   (e) $\displaystyle\int_{-1}^{1} \frac{1}{x^3}\,dx$

2. (15 pts.) Set up the integral for the volume of the solid obtained by rotating the region bounded by

   $$y = 0, \quad y = \sin x, \quad 0 \le x \le \pi$$

   about the line $y = 1$.

   Your answer should include a sketch of the region together with the line about which the region is being rotated.

3. (30 pts.) I have a function $f(x)$ and know that

   $$+4 \le f'(x) \le +20 \quad \text{for} \quad 0 \le x \le 5 \quad \text{and}$$
   $$-3 \le f''(x) \le -2 \quad \text{for} \quad 0 \le x \le 5.$$

   I want to use either the Midpoint rule or the Trapezoidal rule to obtain a lower bound for $\int_0^5 f(x)\,dx$; that is, an estimate which is smaller than $\int_0^5 f(x)\,dx$.

   (a) Which should I use (Midpoint or Trapezoidal) and *why*?
   Hint: A sketch of $f(x)$ may help you find the answer.

   (b) I would like guarantee that the error in my estimate is no larger than $0.001 = 10^{-3}$. How large must I make n to guarantee this?

END OF EXAM