

- Please put your name, ID number, and section number (or time) on your blue book.
  - The exam is CLOSED BOOK, but you may use a page of notes.
  - Calculators are NOT allowed.
  - **You must show your work to receive credit.**
- 
- If you email me your name, section and ID number, I'll reply with your grade.

1. (20 pts.) Evaluate the following. Remember to show your work!

(a)  $\lim_{x \rightarrow 0} \frac{e^x - 1}{\sin x}$       (b)  $(1 + i)^{30}$  where  $i = \sqrt{-1}$

2. (60 pts.) Evaluate the following. Remember to show your work!

(a)  $\int \frac{2}{x^3 - x} dx$       (b)  $\int_1^e \ln x dx$

(c)  $\int \sin^4 x \cos^3 x dx$       (d)  $\int \frac{1}{t\sqrt{t^2 - 1}} dt$

3. (15 pts.) Find the three cube roots of  $1 + i$ . You may leave sines and cosines in your answer, but not inverse trig functions.

4. (20 pts.) The region bounded by  $y^2 = 4x + 4$ ,  $x = 8$ , the  $x$ -axis and the  $y$ -axis is rotated about the  $x$ -axis. Write down integrals for the volume and surface area.

**You do not need to evaluate the integrals.**

5. (15 pts.) Solve the differential equation  $xe^{-t} \frac{dx}{dt} = t$  with the initial condition  $x(0) = 1$ .

6. (20 pts.) To estimate  $\ln 2$ , I plan to evaluate the integral  $\int_1^2 \frac{dx}{x}$  using the Trapezoidal and Midpoint rules with  $n = 10$ .
- (a) One of these will give an overestimate for  $\ln 2$  and the other will give an underestimate. Which gives which estimate and why?  
**Without a reason, you will receive no credit.**
- (b) I change my mind and decide to use  $n = 20$ . If  $E$  is the error using the Midpoint rule with  $n = 10$ , estimate the error using the Midpoint rule with  $n = 20$ . (Express the answer in terms of  $E$ .)
7. (15 pts.) The equation  $r = 4 - \sin \theta$  describes a curve in polar coordinates which encloses a region. Draw a rough sketch of the region and write down an integral for its area.  
**You do not need to evaluate the integral.**
8. (35 pts.) According to a law of physics, water flows out of a hole in the bottom of a circular cylinder at a rate that is proportional to the square root of depth of the water; that is, the rate of outflow for depth  $y$  is  $K y^{1/2}$  where  $K$  is some constant depending on the size of the hole.
- For a particular cylinder, water starts flowing out when the depth is 25 feet. After two minutes, the depth of water is 16 feet.
- (a) Using the above information, set up a differential equation for  $y(t)$ , the depth of the water in feet  $t$  minutes after the start of the flow. Express the 25 feet and 16 feet information as conditions on  $y(t)$  for particular  $t$ .
- (b) **Derive** the formula  $y(t) = (5 - t/2)^2$  by solving the differential equation.  
*NOTE:* "Derive" means you must **actually solve** the differential equation, not simply check that the given  $y(t)$  is a solution.
- (c) What is the depth of water after 10 minutes? after 20 minutes?