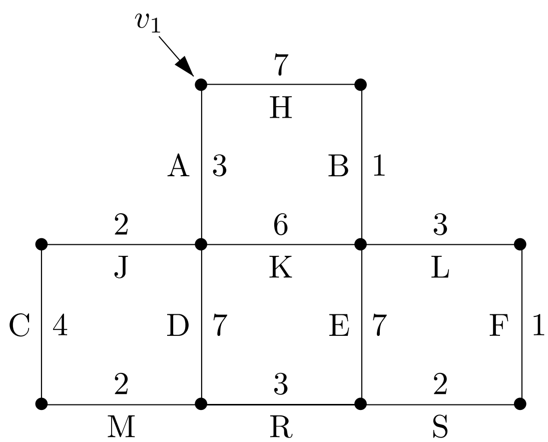
Name ______________________________ ID No. ______________

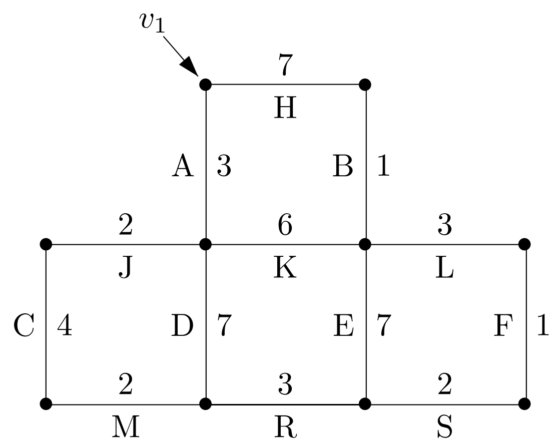
There are 210 points possible.

1. (30 pts.) Recall that Dijkstra's algorithm finds shortest paths from v_1 to all other vertices by adding edges linking in the closest points. In the graph shown below, each edge is bidirectional; that is, you can travel in either direction on it. **Edges are labeled with upper case letters**. (Two copies of the graph are provided so you can use one as a "worksheet" if you wish.)

   (a) List edges in order chosen by algorithm: ______________________________

   (b) At each vertex, give the length of the shortest path from v_1 to the vertex. *Indicate which graph has your answer.*





2. (25 pts.) Consider the following eight complexity categories (remember $\lg = \log_2$):

$$\Theta(n) \quad \Theta(n^2) \quad \Theta(2^n) \quad \Theta(3^{\lg n}) \quad \Theta(n^{\lg n}) \quad \Theta(n \lg n) \quad \Theta((\sqrt{n} + \ln n)^2) \quad \Theta(2^{n+\lg n}).$$

   (a) Which are equal?

   (b) Arrange the distinct classes in order from slowest growing to fastest growing. In other words, if $\Theta(f(n))$ is to the left of $\Theta(g(n))$, then $f(n) \in o(g(n))$.

3. (30 pts.) The average running time for an algorithm is a nondecreasing function of n and satisifies $T(4n) = T(2n) + 2T(n)$ for all $n > 0$. Furthermore, $T(1) = 1$ and $T(2) = 3$.
   (a) Determine $T(2^k)$ as a function of the integer k.
   Hint: Set $t_k = T(2^k)$.

   (b) Determine the complexity class of $T(n)$.

4. (30 pts.) Suppose we have two sorted lists $a_1, \ldots, a_n$ and $b_1, \ldots, b_n$, both of length n, that we want to merge to obtain a sorted list of length $2n$, say $c_1, \ldots, c_{2n}$. To do this, we must decide where the a_i's fit among the b_j's to produce the c list. The number of choices for this is $\binom{2n}{n} \geq 4^n/(2\,n^{1/2})$.

   Suppose the merge is done comparisons of keys. Using the above information, derive a lower bound for the worst case number of key comparisons that are needed. Explain your reasoning; don't just give an answer.

5. (30 pts.) Here is an informal description of a routine `Proc` that is looking for x in a sorted list S. The parameters are the ends of the list. While it is looking it does some processing in `ProcLow` and `ProcHigh`.

```
Proc(lo,hi)
    If lo > hi we are done.
    k = ⌊(lo + hi)/2⌋.
    If S[k] = x, we are done.
    If S[k] < x
        Call ProcHigh(k,hi) and Proc(k + 1,hi)
    Else
        Call ProcLow(lo,k) and Proc(lo,k − 1)
    Endif.
End
```

We begin by calling `Proc(1,n)`. Most of the time is spent in `ProcLow` and `ProcHigh`. In fact, `ProcLow(a,b)` requires $\lg(b-a+1)$ basic operations and `ProcHigh(a,b)` requires $(b-a+1)$ basic operations. (You do *not* need to know what any of this code is supposed to do.)

(a) Let $W(n)$ be the worst case running time for `Proc(1,n)`. Give a recursion and initial condition for $W(2^n)$. (In the worst case, x is not in the list.)

(b) Let $A(n)$ be the average running time for `Proc(1, n)`. Assuming x is not in the list and the probability that $S[k] < x$ is $1/2$, give a recursion for $A(n)$. You need *not* give an initial condition.

6. (65 pts.) Indicate whether true or false. Beware of guessing:

correct answer +5pts. incorrect answer −3pts. no answer 0pts

- ____ $\Theta(2^{n+2}) = \Theta(2^n)$.
- ____ $\Theta((n+2)^2) = \Theta(n^2)$.
- ____ $\Theta(2^{n+\lg n}) = \Theta(2^n)$.
- ____ $\Theta((n+\lg n)^2) = \Theta(n^2)$.
- ____ Greedy algorithms are called "greedy" because they make the best choice at the present time, without concern for the future.
- ____ Dynamic programming algorithms use a bottom up approach.
- ____ Divide and conquer algorithms use a bottom up approach.
- ____ If a divide an conquer algorithm requires recomputing the same quantity many times, it is a good idea to look for a dynamic programming algorithm.
- ____ No greedy algorithm is known for the 0-1 Knapsack Problem.
- ____ It is usually fairly easy to determine average and worst-case time complexities for backtracking algorithms.
- ____ There is a search algorithm that uses comparison of keys and is significantly faster on average and in the worst case than binary search.
- ____ There is a sorting algorithm that uses comparison of keys and is significantly faster on average than and in the worst case than mergesort.
- ____ Quicksort has a good average run time and a poor worst-case run time.