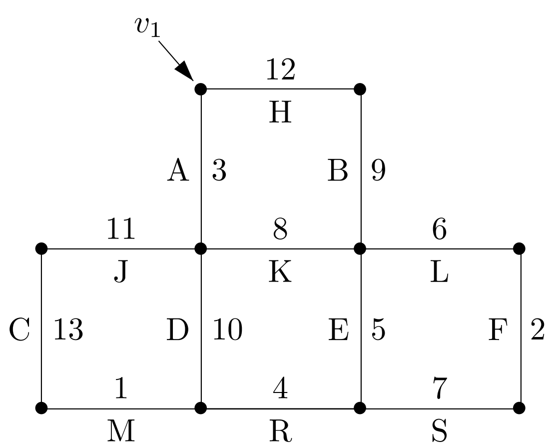
SOLUTIONS TO THE EXAM

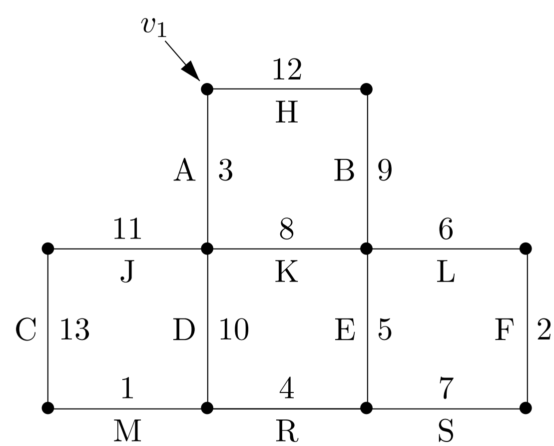
There are 115 points total (So first exam is about 20% and this is about 25%.)

1. (25 pts.) Recall that Prim's algorithm finds a minimum spanning tree by greedily growing a tree starting with v_1, whereas Kruskal's algorithm greedily adds edges in a way that avoids cycles. For the graph shown below, list the edges in the order they are chosen by each algorithm. **Edges are labeled with upper case letters**. (Two copies of the graph are provided so you can use them as "worksheets" if you wish to.)

   (a) Prim's algorithm: A K E R M L F B J

   (b) Kruskal's algorithm: M F A R E L K B J





2. (25 pts.) The worst-case running time for an algorithm is an increasing function of n and satisfies $T(n) = 3T(n/2) + 2n$ when n is a power of two. Furthermore, $T(1) = 1$. Determine the complexity class of $T(n)$.

Ans. Apply Theorem B.5 on page 492 (or Theorem B.6): $a = 3$, $b = 2$, $c = 2$, and $k = 1$. Hence $a > b^k = 2$ and so $T(n) \in \Theta(n^{\log_2 3}) = \Theta(n^{\lg 3})$.

3. (25 pts.) Problem 3.33 says "...write an algorithm to find the maximum sum in any contiguous sublist of a given list of n real numbers. Analyze your algorithm, and show the results using order notation." We present an algorithm below. **Analyze it**. You should **give both average-case and worst-case** complexity information.

```
MaxSum(list, n)
    best = 0    // Best sum so far
    right = 0    // Best sum ending on the end right of 1···i
    For i = 1 to n    // i is the right end
        right = right + list[i]    // Extend sum to the right
        If (right > best)   best = right
        If (right < 0)   right = 0    // Empty sum is better
    End for
End
```

Ans. The basic operation can be anything inside the loop, including the incrementing of i required for the loop. Since the loop is executed n times, the every-case time complexity is $\Theta(n)$. Since this is every-case, it is also average-case and worst-case.

4. (40 pts.) Indicate whether true or false. Beware of guessing:

correct answer +5pts. incorrect answer −3pts. no answer 0pts

(a) F Greedy algorithms are called "greedy" because they often require a lot of storage.

(b) F Dynamic programming algorithms usually split the problem into a few smaller problems, which are solved by recursive calls.

(c) F Usually it is easier to prove that a greedy algorithm is correct than it is to prove that a dynamic programming algorithm is correct.

(d) F If we find a good dynamic programming algorithm for a problem, there will probably not be a good greedy algorithm.

(e) T The "principle of optimality" is a good method for proving that a dynamic programming algorithm is correct.

(f) T A dynamic programming approach is better than a divide and conquer approach for solving a recursion such as $S(n,k) = S(n-1,k) + (k-1)S(n-1,k-1)$. (If $k = 1$ or $n = k$, then $S(n,k) = 1$.)

(g) T Kruskal's algorithm is better than Prim's when the graph has relatively few edges.

(h) F A greedy algorithm for the 0-1 Knapsack Problem is at least as good as a dynamic programming algorithm.

END