Name ______________________________ ID No. ____________

1. (30 pts.) Indicate whether true or false. Beware of guessing:

   correct answer +5pts. incorrect answer −3pts. no answer 0pts

   (a) ____ $(\ln n)^2 \in \Theta(n)$.
   (b) ____ $n \ln n \in o(n^{1.2})$.
   (c) ____ $n^{1.2} \in o(n \ln n)$.
   (d) ____ If $f(n) \in \Theta(g(n))$, then $g(n) \in \Theta(f(n))$.
   (e) ____ If $f_1(n) \in O(g(n))$ and $f_2(n) \in O(g(n))$, then $(f_1(n) + f_2(n)) \in O(g(n))$.
   (f) ____ Let $W_M(n)$ and $W_Q(n)$ be the worst case times for mergesort and quicksort, respectively. True or false: $W_M(n) \in o(W_Q(n))$.

2. (25 pts.) Consider the following eight complexity categories (remember $\lg = \log_2$):

   $$\Theta(2^{\ln n}) \quad \Theta(2^{\lg n}) \quad \Theta(n \lg(\lg n)) \quad \Theta(n \lg n) \quad \Theta(n(1{+}\lg n)) \quad \Theta(n!) \quad \Theta(2^n) \quad \Theta(n)$$

   (a) Which are equal? (There may be more than one pair.) Give a reason for any equalities.

   (b) Arrange the distinct categories in order from slowest growing to fastest growing. In other words, if $\Theta(f(n))$ is to the left of $\Theta(g(n))$, then $f(n) \in o(g(n))$.

3. (20 pts.) It is known that $T(1) = 0$ and that $T(n+1) = 7T(n) + 12$ for $n > 0$. Prove that $T(n) = 2(7^{n-1} - 1)$.

MORE

4. (25 pts.) In the following algorithm, $\cdots$ stands for some simple calculations that take constant time.

```
procedure(n)
    for k from 1 to n do
        ···  /* produces a number j */
        if k divides j, then mergesort an n-long list
        ...
    end for loop
    ...
end
```

Note: Think of j as a random integer, so the probability that "k divides j" is $1/k$.

(a) Suppose the sorting were free (which it is not). What is the complexity class for the average running time of this algorithm. **You MUST give a reason for your answer**. (The class should be of the form $\Theta(f(n))$ where $f(n)$ is a simple function.)

(b) Suppose that the basic operation is a comparison in mergesort. What is the complexity class for the average running time of this algorithm. (You may give your answer in the form $\Theta(\sum f(k))$ where $f(k)$ is a simple function and the sum runs from 1 to n.) **You MUST give a reason for your answer**.

(c) Use (a) and (b) to find the complexity class for the average running time of this algorithm. **You MUST give a reason for your answer**.

END