Math 184A Second Midterm 3/10/99

Each problem is worth 12 points. Please start each problem on a new page.

1. Find a minimum weight spanning tree for the graph shown below. Vertices are indicated by letters and weights of edges are given numerically next to the edges.

   The graph was drawn by hand and is not available.

2. For $k > 0$, let $p_k(n)$ be the number of partitions of the integer n into at most k nonzero parts. Define $p_k(0) = 1$ For example, $p_3(6) = 7$ because

$$6 = 1 + 5 = 2 + 4 = 3 + 3 = 1 + 1 + 4 = 1 + 2 + 3 = 2 + 2 + 2.$$

   (a) Prove that $p_k(n) = \begin{cases} p_{k-1}(n), & \text{if } n < k, \\ p_{k-1}(n) + p_k(n-k), & \text{if } n \geq k. \end{cases}$

   Hint: If there are k nonzero parts, decrease each part by 1.

   (b) Using the result in (a) and the fact that $p_1(n) = 1$ for $n \geq 0$, prove by induction that

$$p_2(n) = \begin{cases} \dfrac{n+2}{2}, & \text{if } n \text{ is even}, \\ \dfrac{n+1}{2}, & \text{if } n \text{ is odd}. \end{cases}$$

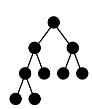
   Hint: Do $n = 0$ and $n = 1$ separately before starting the induction.

3. Consider the (strictly) decreasing functions from $\underline{3}$ to $\underline{11}$.

   (a) Show that there are 165 of them.

   (b) If these functions are arranged in lex order, find the function that is in the exact middle of the list.

4. (a) Find the 7-leaf binary RP-tree whose rank is 77.

   (b) Find the rank of the tree 

END **OF** **EXAM**