

1. (30 pts) (a) Find the strictly decreasing function $f : \underline{4} \rightarrow \underline{20}$ of rank 50.
 (b) Determine the lex order rank of the strictly decreasing function 7, 4, 2, 1. Show your work.
2. (15 pts) Find the 6-leaf binary (unlabeled rooted plane) tree whose rank is 20. Show your work.
3. (55 pts) Define a *web* recursively to be either
 - (i) The simple graph with

$$V = \underline{3} = \{1, 2, 3\} \quad \text{and} \quad E = \left\{ \{1, 2\}, \{1, 3\}, \{2, 3\} \right\}$$

- (a “triangle”) or
 - (ii) A simple graph with $V = \underline{n}$ for some $n > 3$ such that vertex n has degree 2 and removing n and the two edges joining it gives a web with $n - 1$ vertices.
- Do the following.

- (a) Draw the 3 webs that have 4 vertices.
- (b) Prove that a web is not a tree.
- (c) Let w_n be the number of webs with vertex set \underline{n} . Write down a recursion for w_n and explain how you got it.
- (d) Prove that, for $n \geq 3$ the number of webs with vertex set \underline{n} is $\frac{(n-1)!(n-2)!}{2^{n-2}}$.

Here are some values of b_n and binomial coefficients.

$$b_1 = 1 \quad b_2 = 1 \quad b_3 = 2 \quad b_4 = 5 \quad b_5 = 14 \quad b_6 = 42.$$

$$\binom{6}{4} = 15 \quad \binom{7}{4} = 35 \quad \binom{8}{4} = 91 \quad \binom{5}{3} = 10 \quad \binom{6}{3} = 20 \quad \binom{7}{3} = 35 \quad \binom{8}{3} = 56$$

$$\binom{4}{2} = 6 \quad \binom{5}{2} = 10 \quad \binom{6}{2} = 15 \quad \binom{7}{2} = 21 \quad \binom{8}{2} = 28 \quad \binom{9}{2} = 36 \quad \binom{10}{2} = 45$$