

1. In each case, the result is a product by the Rule of Product.
  - (a) Choose the face values for the three of a kind  $\binom{13}{2}$   
 AND the suits for those cards  $\binom{4}{3}^2 = 4^2$ .  
 This gives  $13 \times 6 \times 4^2 = 1248$ .
  - (b) Choose the face values for the pairs  $\binom{13}{2}$   
 AND the suits for those cards  $\binom{4}{2}^2 = 6^2$   
 AND the face values for the remaining two cards  $\binom{11}{2}$   
 AND their suits  $4^2$ .  
 This gives  $13 \times 12 \times 11 \times 10 \times 6^2 \times 4 = 2,471,040$ .
2. (a) IMPOSSIBLE. A list without repeats can't contain more elements than the set.
  - (b) Except for labeling the vertices, there is just one such graph: A square with its diagonal and a fifth point with no edges.
  - (c) IMPOSSIBLE. An  $n$ -vertex simple graph has at most  $\binom{n}{2}$  edges, but  $\binom{4}{2} = 6$ .
3. (a) Since  $\mathcal{P}_2(V)$  has  $N = \binom{n}{2}$  elements, we must form a multiset of size  $q$  from a set of size  $N$ . This can be done in  $\binom{N+q-1}{q}$  ways.
3. (b) Since  $V \times V$  has  $n^2$  elements, we can reason as in (a) with  $N = n^2$ .
4. (a)  $q_k(n) - q_{k-1}(n)$  counts partitions of  $n$  whose largest part is  $k$ . (To see this, note that every partition counted by  $q_k(n)$  is also counted by  $q_{k-1}(n)$  unless it has a part of size  $k$ .)
4. (b) To produce a partition with largest part at most  $k$ , either, *do not* include a part of size  $k$  OR *do* include a part of size  $k$ .
  - *do not*: We have partitions of  $n$  with parts of size at most  $k-1$ , and these are counted by  $q_{k-1}(n)$ .
  - *do*: We have a part of size  $k$  and a partition of what is left into parts of size at most  $k$ . Since  $n-k$  is left, this is counted by  $q_k(n-k)$ .

You were not asked for initial conditions or how large  $n$  and  $k$  must be in the recursion, but here's one way to get an answer. The argument leading to the recursion seems to work for  $n > 0$  and  $k > 1$  if we're careful about what happens when we have a part of size  $k$ . Since that should only happen if  $k \geq n$ ,  $q_k(n-k)$  should be zero when  $k < n$ . In other words,  $q_k(m) = 0$  for  $m < 0$  is one initial condition and  $q_k(0) = 1$  is another. We also need to specify  $q_1(n)$  since that appears in the recursion when  $n = 2$ . Clearly  $q_1(n) = 1$  for  $n > 0$ . These are the initial conditions that we need.