1. Calculate the number of 6 card hands that contain
   (a) two 3-of-a-kind; e.g., 3 eights and 3 jacks;
   (b) two pair, but not three pair or 3 of a kind.

2. Give an example of each of the following, or explain why no such example can exist.
   (a) An ordered list of length 5 with *no repeats* chosen from a set of 4 elements.
   (b) A 5 vertex, 5 edge simple graph that is not connected.
   (c) A 4 vertex, 10 edge simple graph that is connected.

3. Recall that a *simple graph* with vertices V is (V, E) where the edges E are a *set* chosen from $\mathcal{P}_2(V)$ and a simple directed graph (V, E) has edges chosen form $V \times V$.
   (a) A *simple multigraph* with vertices V is (V, E) where the edges E are a *multiset* chosen from $\mathcal{P}_2(V)$. Prove that the number of simple multigraphs with vertices $V = \{1, \ldots, n\}$ and q edges is $\binom{N+q-1}{q}$ where $N = \binom{n}{2}$.
   (b) A *simple directed multigraph* with vertices V is (V, E) where the edges E are a *multiset* chosen from $V \times V$. Find and prove a formula for the number of simple directed multigraphs with vertices $V = \{1, 2, \ldots, n\}$ and q edges.

4. A partition of a positive integer is an unordered list of positive integers whose sum is the given integer. For example, the 7 partitions of 5 are

   $$1+1+1+1+1, \quad 2+1+1+1, \quad 2+2+1, \quad 3+1+1, \quad 3+2, \quad 4+1, \quad 5.$$

   Let $q_k(n)$ be the number of partitions of n whose largest summand is *at most* k. For example,

   $$q_1(5) = 1, \quad q_2(5) = 3, \quad q_3(5) = 5, \quad q_4(5) = 6, \quad q_5(5) = 7.$$

   It is easily seen that $q_1(n) = 1$ for all $n \geq 1$ since the only partition with largest summand at most 1 is $1 + \cdots + 1$.
   (a) Give as *simple as possible* description of the set of partitions counted by $q_k(n) - q_{k-1}(n)$. Your description should refer to the largest part in a partition.
   (b) Prove the recursion $q_k(n) = q_{k-1}(n) + q_k(n-k)$. You need *not* give initial conditions or state how large n and k must be in the recursion.
   (With the initial conditions $q_k(0) = 1$ and $q_k(n) = 0$ for $n < 0$, the recursion holds for $n \geq 1$.)