

1. (a) The solution is not unique. You could construct a two level decision tree based on (i) whether the 4th card with the face value the triple is in the hand (Y, N) and (ii) the number of cards with the face value of the pair in the hand (0, 1, 2). If the choice in (ii) is 2, then the pair is determined, otherwise not. (Another possible tree: reverse the order of (i) and (ii).)

(b) Here are the calculations for the six leaves of the tree in (a).

Y0 (one single card is determined) $11 \times \binom{4}{2} \times \binom{10}{2} \times 4^2 = 47,520$. This comes from (pair face value) \times (pair suits) \times (other single face values) \times (their suits).

Y1 (two single cards determined, but one needs suit) $2 \times 11 \times \binom{4}{2} \times 10 \times 4 = 5,280$.

Y2 (pair and single card determined) $\binom{11}{2} \times 4^2 = 880$.

N0 (no cards in common) $11 \times \binom{4}{2} \times \binom{10}{3} \times 4^3 = 570,240$.

N1 (single card determined, but needs suit) $2 \times 11 \times \binom{4}{2} \times \binom{10}{2} \times 4^2 = 95,040$.

N2 (pair determined) $\binom{11}{3} \times 4^3 = 10,560$.

2. (a) Apply the greedy algorithm to Theorem 3.4 on page 75:

$$200 = \binom{10-1}{4} + \binom{9-1}{3} + \binom{7-1}{2} + \binom{4-1}{1}, \text{ giving the function } 10,9,7,4.$$

(b) By the same formula, the rank is $\binom{8}{5} + \binom{5}{4} + \binom{3}{3} + \binom{1}{2} + \binom{0}{1} = 56 + 5 + 1 = 62$.

3. The order in which the tree is constructed, but not the final tree depend on the algorithm you use. The algorithm on p.153 has an arbitrary feature, namely the choice of v_0 , and this effects the order of construction. If I take $v_0 = A$, the tree edges are selected in the following order:

AB(9) BD(1) BF(2) CD(3) BE(6).

Another algorithm is mentioned in the middle of p.154, was discussed in class, and is proved in Exercise 6.1.6. It chooses edges in the following order

BD(1) BF(2) CD(3) BE(6) AB(9).

4. Suppose that the minimum weight spanning tree does not contain e_1 . Adding e_1 to the tree creates a graph with a cycle and the cycle contains e_1 . Since the cycle contains e_1 , it passes through v and so must contain some edge e_j ($j \neq 1$) that has v as an end. Removing e_j creates a spanning tree of lower weight (since e_1 replaced e_j), a contradiction.