Math 184A Second Hour Exam 4 March 1998

1. We want to know how many pairs of 5-card hands there are such that one hand is a full house (a pair and a triple) and the other hand contains one pair (and three unmatched cards). According to the text (p. 20), there are 3,744 full houses. Let the first hand be a full house.

   (a) (4 pts.) Construct a decision tree to break the possibilities for the second hand into simple cases.

   (b) (8 pts.) Compute the number of hands at any *four* (4) of the leaves in your decision tree from (a). You may leave your answers as products such as $13 \times \binom{4}{2}^3 \times 12$.

2. (a) (6 pts.) Determine the lex order strictly decreasing function $f : \underline{4} \to \underline{12}$ whose rank is 200.

   (b) (6 pts.) Compute the lex order rank of the strictly decreasing function $f : \underline{5} \to \underline{12}$ given by $9, 6, 4, 2, 1$.

3. (6 pts.) Using an algorithm, find the minimum weight spanning tree of the graph shown below. To show that you are using an algorithm, (i) identify it by either giving its location in the tex or describing it and (ii) list the order in which the algorithm selects the edges (and/or vertices) that form the spanning tree.

4. (6 pts.) Suppose the edges of a connected graph G all have different weights. Let $e_1, \ldots, e_d$ be the edges of G listed in *increasing* order of weights. Prove that the minimum weight spanning tree of G contains e_1. Suggestion: Give a proof by contradiction.