1. Of course, there are an infinite number of possible examples. The smallest simple graph with the desired property is given by

$$V = 1, 2, 3, 4 \quad \text{and} \quad E = \big\{\{1,2\},\ \{2,3\},\ \{3,4\},\ \{4,1\},\ \{1,3\}\big\}.$$

It has $v = |V| = 4$ and $|E| = 5 = v + 1$, and it contains the three cycles whose vertices (in order) are (a) 1,2,3,4, (b) 1,2,3, and (c) 1,4,3.

For those who may be interested, it is well known in graph theory circles that a simple graph with v vertices and $v+k-1$ edges contains at most $2^k - 1$ cycles. In fact, such a graph contains precisely that many "generalized" cycles, where a generalized cycle is a nonempty union of edge-disjoint cycles.

2. (a) The answer is 43. Here's one way to do it: Consider cases according to the number of repeated letters:
   - For no repeated letters, there are $4 \times 3 \times 2 = 24$ possibilities.
   - For two repeated letters, choose the repeat from A and L, AND choose the other letter from the three remaining, AND choose the position for that letter, giving $2 \times 3 \times 3 = 18$.
   - The only way to get a letter 3 times is with LLL.

   (b) The answer is 420. Here's one way to do it: If we distinguish among the 3 L's and 2 A's, each original list gives rise to $3! \times 2! = 12$ lists because of the ways to label the L's (L_1, L_2, and L_3, say) and the A's. Since the number of lists will all letters distinct is $7! = 5040$, the answer is $5040/12 = 420$.

   Here's another way to do it: Choose a position for J, AND, from the remaining 6 positions, choose one for O, AND, from the remaining 5 positions, choose 2 positions for the A's. This gives $7 \times 6 \times \binom{5}{2} = 420$ ways.

3. (a) Here's the computation:

$$\begin{aligned}
p_2(2) &= 1 \quad \text{from} \quad 1+1 \\
p_2(3) &= 1 \quad \text{from} \quad 2+1 \\
p_2(4) &= 2 \quad \text{from} \quad 3+1,\ 2+2 \\
p_2(5) &= 2 \quad \text{from} \quad 4+1,\ 3+2 \\
p_2(6) &= 3 \quad \text{from} \quad 5+1,\ 4+2,\ 3+3 \\
p_2(7) &= 3 \quad \text{from} \quad 6+1,\ 5+2,\ 4+3
\end{aligned}$$

It looks like $p_2(n)$ is $n/2$ rounded down. In other words, $p_2(n) = [n/2]$, where $[x]$ is the largest integer not exceeding x. In still other words,

$$p_2(n) = \begin{cases} n/2 & \text{if } n \text{ is even,} \\ (n-1)/2 & \text{if } n \text{ is odd.} \end{cases}$$

(b) Let $n = j + k$ be a partition of n. If $j \geq k$, then $n \geq k + k = 2k$ and so $k \leq n/2$. On the other hand, if $k \leq n/2$, then $j = n - k \geq n/2$ and so $j \geq k$. We have just shown that $j \geq k$ if and only if $k \leq n/2$. Thus k can be any of the positive integers that do not exceed $n/2$, which yields the formula given in (a).