1. (6 pts.) Recall that a cycle of a simple graph is a subgraph consisting of some set $\{v_1, \ldots, v_k\}$ of vertices together with the k edges $\{v_1, v_2\}$, $\{v_2, v_3\}$, …, $\{v_k, v_1\}$.
   - It follows from Exercise 5.5.3 that a connected v-vertex graph that has at least one cycle has at least v edges. (You are not asked to do that.)
   - It can be shown that a connected v-vertex graph that has at least two cycles has at least $v+1$ edges. (You are not asked to do that.)

   One might expect the pattern to continue: at least k cycles implies at least $v+k-1$ edges.
   Show that this doesn't happen by exhibiting for some v a connected v-vertex simple graph with more than two cycles and only $v+1$ edges. Be sure to describe the cycles!

2. (a) (8 pts.) How many ways are there to form an *ordered/* list of 3 (three) letters from the letters in LAJOLLA (3 L's, 2 A's, 1 J, and 1 O), provided no letter can be used more often than it appears in LAJOLLA?

   (b) (8 pts.) Repeat the above for 7 (seven) letters (i.e., use all the letters).

3. A partition of a positive integer is an unordered list of positive integers whose sum is the given integer. For example, the 7 partitions of 5 are

$$5, \quad 4+1, \quad 3+2, \quad 3+1+1, \quad 2+2+1, \quad 2+1+1+1, \quad 1+1+1+1+1.$$

   Let $p_k(n)$ be the number of partitions of n with exactly k summands. For example,

$$p_1(5) = 1, \quad p_2(5) = 2, \quad p_3(5) = 2, \quad p_4(5) = 1, \quad p_5(5) = 1.$$

   (a) (6 pts.) Compute $p_2(n)$ for $2 \le n \le 7$. Use this to conjecture a formula for $p_2(n)$ for all n.

   (b) (8 pts.) Prove the formula for $p_2(n)$ that was conjectured in (a).