

1. (a) One solution is to form a first block and a second block and divide by two because the blocks shouldn't be labeled. The first block can be any of the 2^n subsets except the empty set and the entire set since these would give an empty block. Thus we get $(2^n - 2)/2$.
Another solution is to fix one element of the set and choose the other block. We can use any of the 2^{n-1} except the empty set. Thus we get $2^{n-1} - 1$.
- (b) To get $n - 1$ blocks we must have a 2-block and $n - 2$ 1-blocks. There are $\binom{n}{2}$ ways to form the 2-block.
2. Starting at a fixed point, read around the table in, say, the clockwise direction. People must alternate in sex. Other than that, they can be arranged in any manner. So, choose the starting sex, choose a permutation for the men, and choose a permutation for the women. We get $2(n!)^2$.
3. (a) With 10 edges, there can be at most one repeated weight. Thus there could be as many as two minimum weight spanning trees.
- (b) With 11 edges, we could have one weight repeated 3 times or two weights repeated twice. The first case gives a maximum of 3 trees. The second gives a maximum of 2×2 , so the answer to (b) is 4.
4. We use Inclusion-Exclusion. Let S_i be the colorings that omit the i^{th} color. This leads to the general answer for n colors of $\sum_{t=0}^n (-1)^t \binom{n}{t} P_G(n-t)$. Specialize to $n = 6$.
5. (a) Let w_n be the amount of work. From the local description, $w_1 = 1$ and $w_n = 2w_{n-1} + n^2$ for $n > 1$.
- (b) The formula gives $12 - 1 - 4 - 6 = 1$ when $n > 1$. When $n > 1$,
- $$2(6 \times 2^{n-1} - (n-1)^2 - 4(n-1) - 6) + n^2 = 6 \times 2^n - n^2 - 4n - 6.$$
- Thus the formula satisfies the initial condition and the recursion.
6. You could do this by listing all the trees before the given tree. I'll use the formula. Note that the left and right children of the root are the same and compute that their rank is 1. We then have $b_1 b_5 + b_2 b_4 + 1 b_3 + 1 = 14 + 5 + 2 + 1 = 22$.
7. From the description

$$T(x) = x + \sum_{k \text{ odd}} x(T(x))^k = x + \sum_{n=0}^{\infty} xT(x)(T(x)^2)^n = x + \frac{xT(x)}{1 - T(x)^2}.$$

Clearing of fractions, $T(x) - T(x)^3 = x - xT(x)^2 + xT(x)$. Rearrange.

8. We use Principle 11.7 with $F(x, y) = xe^y - y$. Thus $F_y = xe^y - 1$, $F_x = e^y$ and $F_{yy} = xe^y$. To get r and s we solve

$$0 = F(r, s) = re^s - s \quad \text{and} \quad 0 = F_y(r, s) = re^s - 1.$$

The solution is $s = 1$ and $r = 1/e$. Thus $F_x(r, s) = e$, $F_{yy}(r, s) = 1$ and so

$$\frac{g_n}{n!} \sim \frac{n^{-3/2}e^n}{\sqrt{2\pi}}.$$