1. See Exercise 1.2.11.

2. See Exercise 1.3.3.

3. One department must contribute two members and the other two must contribute one member each. Breaking this down into three cases, we have

$$\binom{5}{2} \times 7 \times 6 \quad + \quad 5 \times \binom{7}{2} \times 6 \quad + \quad 5 \times 7 \times \binom{6}{2}.$$

   You could, but need not, simplify this to

$$5 \times 7 \times 6 \left(\frac{4}{2} + \frac{6}{2} + \frac{5}{2} \right) \;=\; 5 \times 7 \times 3(4+6+5) \;=\; 5 \times 7 \times 3 \times 15.$$

4. Since the sum of the cycle lengths is 5, the possible multisets of cycle lengths are

$$\{1,1,1,1,1\} \quad \{1,1,1,2\} \quad \{1,1,3\} \quad \{1,2,2\} \quad \{1,4\} \quad \{2,3\} \quad \{5\}.$$

   Since f^k will be the identity if and only if all the cycle lengths divide k, it follows that $\{2,3\}$ is the only case. In other words, those permutations that have one 2-cycle and one 3-cycle.

5. Listing the two elements with the smaller first, we see that the smaller element in a 2-part partition of n can have any positive integer value not exceeding $n/2$.
   (a) When $n = 2m$, the possible values are $1, 2, \ldots, m$, which proves the claim.
   (b) When $n = 2m+1$, the possible values are $1, 2, \ldots, m$ and so the answer is m.