- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK except for one page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (8 pts.) Prove that the number of ordered lists without repeats (including the empty list) that can be constructed from an n-set is nearly $n!\,e$.

   Hint: By Taylor's theorem, e is nearly $1 + 1/1! + 1/2! + 1/3! + \cdots + 1/n!$.

2. (8 pts.) How many 6-card hands contain 3 pairs? As usual:
   - We are assuming a deck of 52 cards with 13 face values and 4 suits.
   - A pair is two cards with the same face values.
   - Three pairs means each pair has a different face value from the other two.

   You may leave factorials and binomial coefficients in your answer.

3. (8 pts.) The departments of Engineering, Mathematics and Computer Science at Small College have 5 (five), 7 (seven) and 6 (six) members respectively. They want to form a 4-person committee that contains at least one person from each department. How many possible committees are there?
   You may leave factorials and binomial coefficients in your answer.

4. (8 pts.) Describe all permutations f of $\{1, 2, 3, 4, 5\}$ such that f^k is *not* the identity for any positive $k \in \{1, 2, 3, 4, 5\}$. You must justify your answer.

   Hint: Look at cycle lengths.

5. (8 pts.) A k-part partition of n is a k-multiset of positive integers whose sum is n. For example the 2-part partitions of 6 are $\{1, 5\}$, $\{2, 4\}$ and $\{3, 3\}$.

   (a) Prove that there are exactly m 2-part partitions of $2m$ when $m > 0$.

   (b) State and prove a formula for the number of 2-part partitions of $2m + 1$ when $m > 0$. *Hint*: If you do not see the formula right away, list the partitions for $m = 1$, $m = 2$ and maybe $m = 3$.

END OF EXAM