1. (a) FALSE [only true for connected graphs]
   (b) TRUE
   (c) TRUE
   (d) FALSE [look at 1 and 3 in Problem 2]
   (e) TRUE [$P_G(10) = 0$ means ten colors are not enough]

2. The graph is three triangles sharing vertex 0. One way to compute $P_G(x)$: first color vertex 0 (x ways); then color vertices 1,3,5 ($(x-1)^3$ ways); then color vertices 2,4,6 ($(x-2)^3$ ways). Hence $P_G(x) = x(x-1)^3(x-2)^3$.

3. (a) Let c be the number of components of G. We are given that $c > 1$. Suppose G does not have a cycle, then each of its components are trees. If n_k is the number of vertices in the kth component, then $\sum_{k=1}^{c} n_k = 25$. Since a tree has one fewer edges than vertices,

$$24 \;=\; (\text{number of edges}) \;=\; \sum_{k=1}^{c}(n_k - 1) \;=\; 25 - c \;\le\; 23,$$

   a contradiction.

   (b) Let G consist of two trees that have a total of 25 vertices. There will be 23 edges.

4. (a) Set $x = \pm r$ and $y = z$ in the hint to obtain

$$\sum_{n,k} \binom{n}{k} r^n z^k \;=\; \frac{1}{1-(1+z)r} \quad\text{and}\quad \sum_{n,k} \binom{n}{k} (-r)^n z^k \;=\; \frac{1}{1+(1+z)r}.$$

   Add and then divide by 2 to obtain the answer.

   (b) We have

$$2\sum_{n=0}^{\infty} \binom{2n}{k} (1/2)^{2n} \;=\; [z^k]\left(\frac{2}{1-z} + \frac{2/3}{1+z/3}\right) \;=\; 2 + (2/3)(-1/3)^k$$

   and so $s_k(1/2) = 1 - (-1/3)^{k+1}$.

5. (a) By the methods in Section 10.4, $T(x) = x + (T(x))^2 + (T(x))^3 + (T(x))^4 + \cdots$. Summing the geometric series, you rewrite this as $T(x) = x + \frac{T(x)^2}{1-T(x)}$. Either form, or any equivalent form is fine.

   If you incorrectly counted by number of vertices, you would have an extra factor of x and would receive partial credit for it: $T(x) = x + \frac{xT(x)^2}{1-T(x)}$.

   (b) From$T = x + \frac{T^2}{1-T}$ we have $2T^2 - T + x = 0$. Thus

$$T(x) \;=\; \frac{1 \pm \sqrt{1-8x}}{4}.$$

Since $T(0) = 0$, the minus sign is the correct choice.

If you got the wrong answer in (a) but solved your wrong answer correctly in (b), you'll receive credit. For example, counting by vertices gives $(1+x)T^2 - T + x = 0$ and so

$$T(x) = \frac{1 \pm \sqrt{1 - 4x(1+x)}}{2(1+x)}, \quad \text{and the minus sign is correct.}$$