

1. See Exercise 1.2.11.
2. (a) Impossible since a function from a 3-set to a 4-set cannot hit everything in the 4-set.
(b) There are 24 examples. One is $f(1) = a, f(2) = b, f(3) = c$.
(c) Any permutation with a 3-cycle and a 2-cycle will work. In this case f^{40} is the 3-cycle of f and two 1-cycles from the elements of the 2-cycle of f . For example, $f = (1, 2, 3)(4, 5)$ and $f^{40} = (1, 2, 3)(4)(5)$, which can also be written $(1, 2, 3)$. (If f has no 3-cycles, the only possible cycle lengths divide 40 and so f^{40} is the identity. If f had a 3-cycle and two 1-cycles, then $f^{40} = f$.)
3. (a) Let the number of leaves be s_n . From the statement of the problem, $s_1 = 2$ and $s_2 = 3$. From the third figure in the exercise, there are s_{n-1} leaves on the left and s_{n-2} on the right and so $s_n = s_{n-1} + s_{n-2}$ when $n \geq 3$.
(b) Use induction. True for $n = 1$ and $n = 2$ by inspection. For $n > 2$, the left (resp. right) tree has leaves of length $1 + (n - 1) = n$ (resp. $2 + (n - 2) = n$).
4. Listing the two elements with the smaller first, we see that the smaller element in a 2-part partition of n can have any positive integer value not exceeding $n/2$.
(a) When $n = 2m$, the possible values are $1, 2, \dots, m$, which proves the claim.
(b) When $n = 2m + 1$, the possible values are $1, 2, \dots, m$ and so the answer is m .