Math 184A | First Exam (40 points) | 19 October 2005

- Please put your name and ID number on your blue book.
- The exam is CLOSED BOOK except for one page of notes.
- Calculators are NOT allowed.
- **You must show your work to receive credit.**

1. (10 pts.) Prove that the number of ordered lists without repeats (including the empty list) that can be constructed from an n-set is nearly $n!\,e$.

   Hint: By Taylor's theorem, e is nearly $1 + 1/1! + 1/2! + 1/3! + \cdots + 1/n!$.

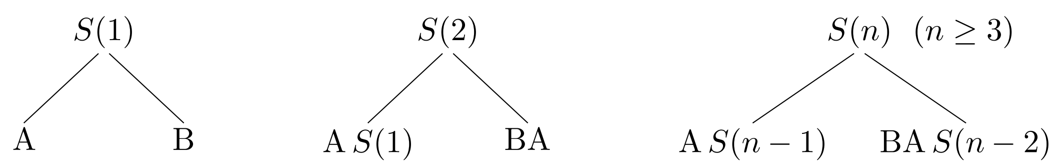
2. (10 pts.) For each of the following,
   EITHER give an example of the thing that is described
   OR explain why none exists.

   (a) A surjection from $\{1, 2, 3\}$ to $\{a, b, c, d\}$.

   (b) An injection from $\{1, 2, 3\}$ to $\{a, b, c, d\}$.

   (c) A permutation f of $\{1, 2, 3, 4, 5\}$ such that f^{40} is NOT the identity function and $f^{40} \neq f$. Also, if you find such an f, compute f^{40}.
   Remember that the identity function is the function g such that $g(x) = x$ for all x and $f^{40}(x)$ is $f(f(\cdots f(x)))$, not $(f(x))^{40}$.

3. (10 pts.) The local description of a decision tree for constructing sequences of A's and B's is given below. The notation BA $S(n-2)$ means place BA in front of each sequence produced by $S(n-2)$.



   Let $S^*(n)$ denote the entire decision tree. Thus $S^*(1) = S(1)$ and $S^*(2)$ has the three leaves AA, AB, and BA.

   (a) Obtain a recursion, with initial conditions for the number of leaves of $S^*(n)$.
   To obtain credit, you must explain how you got the recursion.

   (b) Prove that each leaf of $S^*(n)$ is an n-long sequence of A's and B's.

4. (10 pts.) A k-part partition of n is a k-multiset of positive integers whose sum is n. For example the 2-part partitions of 6 are $\{1, 5\}$, $\{2, 4\}$ and $\{3, 3\}$.

   (a) Prove that there are exactly m 2-part partitions of $2m$ when $m > 0$.

   (b) State and prove a formula for the number of 2-part partitions of $2m + 1$ when $m > 0$. *Hint*: If you do not see the formula right away, list the partitions for $m = 1$, $m = 2$ and maybe $m = 3$.

END OF EXAM