1. (a) There are 9 choices for the leftmost digit AND then 9 for the next AND then 8 for the last, giving $9 \times 9 \times 8$.
   Stating $9 \times 9 \times 8$ without explanation is sufficient, since it is clear that you've thought about it correctly. On the other hand, it is hard to give partial credit to a wrong answer if it has no explanation.

   (b) There are 4 patterns of E (even) and O (odd) that have an odd sum. Remembering that 0 cannot be the first digit and digits must differ, here are the patterns and their counts.

$$EEO:\ 4\times 4\times 5 \quad EOE:\ 4\times 5\times 4 \quad OEE:\ 5\times 5\times 4 \quad OOO:\ 5\times 4\times 3.$$

   The sum of these is the answer. There is no need to carry out the calculations, but if you do, you should get

$$5\times 4\times (4+4+5+3) \;=\; 5\times 4\times 16 \;=\; 320,$$

   which is a bit less than half of (a).

2. Using the formula:

$$\binom{5}{3}+\binom{2}{2}+\binom{0}{1} \;=\; 11.$$

   You could also get this by drawing the decision tree.

3. If $a_k = t$, then the first k numbers in the sequence are a strictly increasing list from $\underline{t-1}$, which is the same as a k-subset of $\underline{t-1}$. There are $\binom{t-1}{k}$ of these. A similar argument works for the last k. Using the rules of sum and product, we obtain

$$P(n,k) \;=\; \sum_{t=1}^{n}\binom{t-1}{k}^2.$$

   (This even works for $k=0$.) If you wish, you could start the sum at $t=k+1$ and you could rewrite the sum as

$$P(n,k) \;=\; \sum_{t=k+1}^{n}\binom{t-1}{k}^2 \;=\; \sum_{s=k}^{n-1}\binom{s}{k}^2.$$

END OF EXAM